

Optimal Delay–Throughput Tradeoffs in Mobile *Ad Hoc* Networks

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Abstract—In this paper, we investigate the delay–throughput tradeoffs in mobile *ad-hoc* networks. We consider four node mobility models: 1) two-dimensional independent and identically distributed (i.i.d.) mobility, 2) two-dimensional hybrid random walk, 3) one-dimensional i.i.d. mobility, and 4) one-dimensional hybrid random walk. Two mobility time scales are included in this paper. i) Fast mobility, where node mobility is at the same time scale as data transmissions. ii) Slow mobility, where node mobility is assumed to occur at a much slower time scale than data transmissions. Given a delay constraint D , we first characterize the maximum throughput per source–destination (S-D) pair for each of the four mobility models with fast or slow mobiles. We then develop joint coding–scheduling algorithms to achieve the optimal delay–throughput tradeoffs.

Index Terms—Delay–throughput tradeoffs, hybrid random walk models, independent and identically distributed (i.i.d.) mobility models, mobile *ad hoc* networks, rate-less codes, scaling laws.

I. NOTATIONS

THE following notations are used throughout this paper. Given nonnegative functions $f(n)$ and $g(n)$:

- (1) $f(n) = O(g(n))$ means there exist positive constants c and m such that $f(n) \leq cg(n)$ for all $n \geq m$;
- (2) $f(n) = \Omega(g(n))$ means there exist positive constants c and m such that $f(n) \geq cg(n)$ for all $n \geq m$; namely, $g(n) = O(f(n))$;
- (3) $f(n) = \Theta(g(n))$ means that both $f(n) = \Omega(g(n))$ and $f(n) = O(g(n))$ hold;
- (4) $f(n) = o(g(n))$ means that $\lim_{n \rightarrow \infty} f(n)/g(n) = 0$;
- (5) $f(n) = \omega(g(n))$ means that $\lim_{n \rightarrow \infty} g(n)/f(n) = 0$; namely, $g(n) = o(f(n))$.

II. INTRODUCTION

In this paper, we investigate the delay–throughput tradeoffs in a mobile *ad hoc* network with n mobiles. The throughput of

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a random wireless network with n static nodes and n random source–destination (S-D) pairs was studied by Gupta and Kumar [1]. They showed that the maximum throughput per S-D pair is $O(1/\sqrt{n})$, and proposed a scheduling scheme achieving a throughput of $\Theta(1/\sqrt{n \log n})$ per S-D pair. One of the reasons that the throughput decreases with n is that each successful transmission from source to destination needs to take $\sqrt{n/\log n}$ hops. Later, Grossglauser and Tse [2] considered mobile *ad hoc* networks, and showed that $\Theta(1)$ throughput per S-D pair is achievable. The idea is to deliver a packet to its destination only when it is within distance $\Theta(1/\sqrt{n})$ from the destination. However, packets have to tolerate large delays to achieve this throughput. Since then, a number of papers have studied the tradeoffs between throughput and delay.

We first review the delay–throughput tradeoff results for independent and identically distributed (i.i.d.) mobility models. Neely and Modiano [3] studied the i.i.d. mobility model where the positions of nodes are totally reshuffled from one time slot to another, and showed that the mean delay of Grossglauser and Tse’s algorithm is $\Theta(n)$. In the same paper, they also proposed an algorithm which generates multiple copies of each data packet to reduce the mean delay. Since more transmissions are required when we generate multiple copies, the throughput per S-D decreases with the number of copies per data packet. The delay–throughput tradeoff is shown to be $\lambda = \Omega(D/n)$ in [3], where λ is the throughput per S-D pair, and D is the number of time slots taken to deliver packets from source to destination.

In [3], fast mobility is assumed. A different time scale of mobility, slow mobility, was considered by Toumpis and Goldsmith in [4], and Lin and Shroff in [5]. For slow mobiles, node mobility is assumed to be much slower than data transmissions. So the packet size can be scaled down as n increases, and multihop transmissions are feasible in single time slot. The delay–throughput tradeoff was shown to be $\lambda = \Omega(\sqrt{D/n \log n})$ in [4]. A better tradeoff was obtained in [5], where the maximum throughput per S-D pair for mean delay D was shown to be $\lambda = O(\sqrt[3]{D/n \log n})$, and a scheme was proposed to achieve a tradeoff of

$$\lambda = \Theta\left(\sqrt[3]{D/(n \log^{9/2} n)}\right).$$

Besides the i.i.d. mobility model, other mobility models have also been studied in the literature. The random walk model was introduced by El Gamal *et al.* in [6], and later studied in [7]–[9]. In [7], [8], the throughput per S-D pair is shown to be $\Theta(1/\sqrt{n \log n})$ for $D = O(\sqrt{n/\log n})$, and $\Theta(D/n)$ for $D = \Omega(\sqrt{n/\log n})$, where [7] focused on the slow mobility and [8] focused on the fast mobility. Other mobility models,

like Brownian motion, one-dimensional mobility, and hybrid random walk models have been studied in [9]–[12].

Although the delay–throughput tradeoff has been widely studied for various mobility models, the optimal delay–throughput tradeoff has not yet been established except for two cases of mobility models [7], [8], [10]. In this paper, we study four different node mobility models. First, we study the two-dimensional i.i.d. mobility model, where the nodes are totally reshuffled at each time slot. Then we extend the results to the two-dimensional hybrid random walk model, which was introduced by Sharma *et al.* in [9]. Under the two-dimensional hybrid random walk model, the unit torus is divided into $1/S^2$ small-squares, and mobiles can move from the current small-square to one of its eight adjacent small-squares at the beginning of each time slot. Since the distance each mobile can move is at most $2\sqrt{2}/S$ at each time slot, we can use different values of S to model mobiles with different speeds. Note that the two-dimensional hybrid random walk model is the same as the two-dimensional i.i.d. mobility model when $S = 1$. However, the Markovian mobility dynamics of the random walk requires a different set of tools and as a result, the tradeoffs of the two-dimensional hybrid random walk model are applicable only when $S = o(1)$.

We also study one-dimensional mobility models. These models are motivated by certain types of delay-tolerant networks [13], in which a satellite subnetwork is used to connect local wireless networks outside of the Internet. Since the satellites move in fixed orbits, they can be modeled as one-dimensional mobilities on a two-dimensional plane. Motivated by such a delay-tolerant network, we consider one-dimensional mobility model where n nodes move horizontally and the other n nodes move vertically. Since the node mobility is restricted to one dimension, sources have more information about the positions of destinations compared with the two-dimensional mobility models. We will see that the throughput is improved in this case; for example, under the one-dimensional i.i.d. mobility model with fast mobiles, the tradeoff will be shown to be $\Theta(\sqrt[3]{D^2/n})$, which is better than $\Theta(\sqrt{D/n})$, the tradeoff under the two-dimensional i.i.d. mobility model with fast mobiles.

We summarize our main results as follows.

- (1) Two-dimensional i.i.d. mobility models:
 - (i) Under the fast mobility assumption, it is shown that the maximum throughput per S-D pair is $O(\sqrt{D/n})$ under a delay constraint D . A joint coding–scheduling algorithm is presented to achieve the maximum throughput when D is both $\omega(\sqrt[3]{n})$ and $o(n)$.
 - (ii) Under the slow mobility assumption, it is shown that the maximum throughput per S-D pair is $O(\sqrt[3]{D/n})$ given a delay constraint D . We propose a joint coding–scheduling algorithm to achieve the maximum throughput when D is both $\omega(1)$ and $o(n)$.
- (2) Two-dimensional hybrid random walk models:
 - (i) Under the fast mobility assumption, it is shown that the maximum throughput per S-D pair is $O(\sqrt{D/n})$ when $S = o(1)$ and $D = \omega(|\log S|/S^2)$, where S is the step size of the random walk. A joint coding–scheduling algorithm is proposed to achieve the maximum throughput when $S = o(1)$ and D is both $\omega(\max\{(\log^2 n)|\log S|/S^6, \sqrt[3]{n}\log n\})$ and $o(n/\log^2 n)$.
 - (ii) Under the slow mobility assumption, it is shown that the maximum throughput per S-D pair is $O(\sqrt[3]{D/n})$ when $S = o(1)$ and $D = \omega(|\log S|/S^2)$, and a joint coding–scheduling algorithm is proposed to achieve the maximum throughput when $S = o(1)$ and D is both $\omega((\log^2 n)|\log S|/S^6)$ and $o(n/\log^2 n)$.
- (3) One-dimensional i.i.d. mobility models:
 - (i) Under the fast mobility assumption, it is shown that the maximum throughput per S-D pair is $O(\sqrt[3]{D^2/n})$ given a delay constraint D . A joint coding–scheduling algorithm is proposed to achieve the maximum throughput when D is both $\omega(\sqrt[5]{n})$ and $o(\sqrt{n}/\sqrt[3]{2\log n})$.
 - (ii) Under the slow mobility assumption, it is shown that the maximum throughput per S-D pair is $O(\sqrt[4]{D^2/n})$. A joint coding–scheduling algorithm is proposed to achieve the maximum throughput when D is $o(\sqrt{n}/\log^2 n)$.
- (4) One-dimensional hybrid random walk models:
 - (i) Under the fast mobility assumption, it is shown that the maximum throughput per S-D pair is $O(\sqrt[3]{D^2/n})$ when $S = o(1)$ and $D = \omega(1/S^2)$, and a joint coding–scheduling algorithm is proposed to achieve the maximum throughput when $S = o(1)$ and D is both $\omega(\max\{(\log^2 n)|\log S|/S^4, \sqrt[5]{n}\log n\})$ and $o(\sqrt{n}/\sqrt[3]{2\log n})$.
 - (ii) Under the slow mobility assumption, it is shown that the maximum throughput per S-D pair is $O(\sqrt[4]{D^2/n})$ when $S = o(1)$ and $D = \omega(1/S^2)$, and a joint coding–scheduling algorithm is proposed to achieve the maximum throughput when $S = o(1)$ and D is both $\omega((\log^2 n)|\log S|/S^4)$ and $o(\sqrt{n}/\log^2 n)$.

Note that the optimal delay–throughput tradeoff are established under some conditions on D . When these conditions are not met, the tradeoff is still unknown in general, though a tradeoff of the two-dimensional hybrid random walk model with slow mobiles has been established under an assumption regarding packet replication in [9]. We also would like to mention that when the step size of the two-dimensional hybrid random walk is $1/\sqrt{n}$, our two-dimensional hybrid random walk model is identical to the random walk model studied in [7], [8], where the optimal delay–throughput tradeoff has been obtained. Our results do not apply to this case since the set of allowed values for D becomes empty in that case (see (2) (i) above).

We also would like to mention that there is a very recent result by Ozgur *et al.* [14] where they showed a throughput of $\Theta(1)$ per S-D pair is achievable using node cooperation and mul-

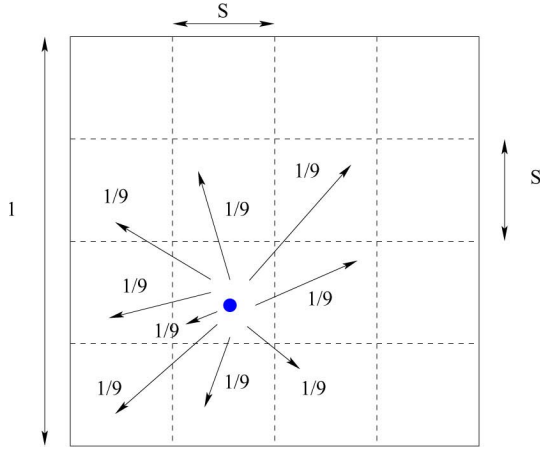


Fig. 1. Two-dimensional random walk model.

multiple-input multiple-output (MIMO) communication; see also the earlier paper by Aeron and Saligrama in [15]. These schemes require sophisticated signal processing techniques, not considered in this paper.

III. MODEL

In this section, we first present the mobility and wireless interference models used in this paper. Then the definitions of delay and throughput are provided.

Mobile *ad hoc* network model: Consider an *ad hoc* network where wireless mobile nodes are positioned in a unit square. The unit square is assumed to be a torus, i.e., the top and bottom edges are assumed to touch each other and similarly the left and right edges also are assumed to touch each other. Further assume that the time is slotted, we study following mobility models in this paper.

- (1) **Two-dimensional i.i.d. mobility model:** Our two-dimensional i.i.d. mobility model is defined as follows.
 - (i) There are n wireless mobile nodes positioned on a unit square. At each time slot, the nodes are uniformly, randomly positioned in the unit square.
 - (ii) The node positions are independent of each other, and independent from time slot to time slot. So the nodes are totally reshuffled at each time slot.
 - (iii) There are n S-D pairs in the network. Each node is both a source and a destination. Without loss of generality, we assume that the destination of node i is node $i + 1$, and the destination of node n is node 1.
- (2) **Two-dimensional random walk model:** Consider a unit square which is further divided into $1/S^2$ squares of equal size. Each of the smaller square will be called an RW-cell (random walk cell), and indexed by (U^x, U^y) where $U^x, U^y \in \{1, \dots, 1/S\}$. A node which is in one RW-cell at a time slot moves to one of its eight adjacent RW-cells or stays in the same RW-cell in the next time slot with each move being equally likely as in Fig. 1. Two RW-cells are said to be adjacent if they share a

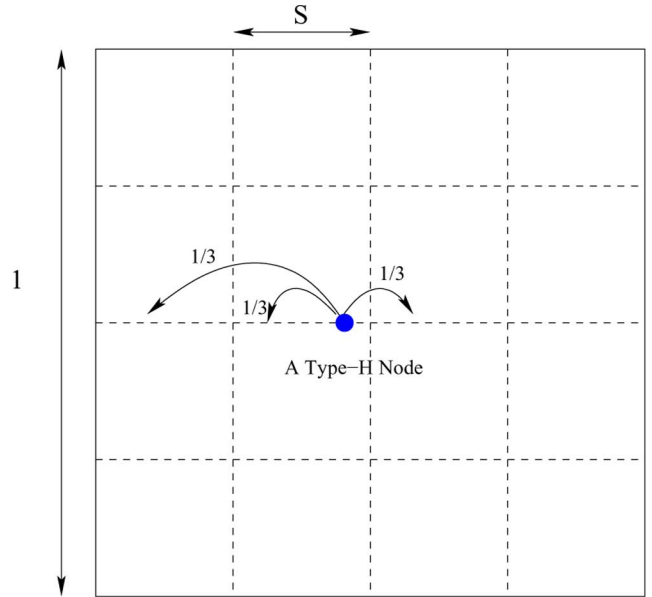


Fig. 2. One-dimensional random walk model.

common point. The node position within the RW-cell is randomly uniformly selected.

Remark: In our joint coding-scheduling algorithms, the unit square will be divided into cells, and only those mobiles in the same cell are allowed to communicate with each other. These cells, used for scheduling and communication, are different from the RW-cells defined above.

- (3) **One-dimensional i.i.d. mobility model:** Our one-dimensional i.i.d. mobility model is defined as follows.
 - (i) There are $2n$ nodes in the network. Among them, n nodes, named H-nodes, move horizontally; and the other n nodes, named V-nodes, move vertically.
 - (ii) Letting (x_i, y_i) denote the position of node i . If node i is an H-node, y_i is fixed and x_i is a value randomly uniformly chosen from $[0, 1]$. We also assume that H-nodes are evenly distributed vertically, so y_i takes values $1/n, 2/n, \dots, 1$. V-nodes have similar properties.
 - (iii) Assume that source and destination are the same type of nodes. Also assume that node i is an H-node if i is odd, and a V-node if i is even. Further, assume that the destination of node i is node $i + 2$, the destination of node $2n - 1$ is node 1, and the destination of node $2n$ is node 2.
 - (iv) The orbit distance of two H(V)-nodes is defined to be the vertical (horizontal) distance of the two nodes.
- (4) **One-dimensional random walk model:** Each orbit is divided into $1/S$ RW-intervals (random walk interval). At each time slot, a node moves into one of two adjacent RW-intervals or stays at the current RW-interval (as in Fig. 2). The node position in the RW-interval is randomly, uniformly selected.

Communication model: We assume the protocol model introduced in [1] in this paper. Let $\text{dist}(i, j)$ denote the Euclidean distance between node i and node j , and r_i denote the transmission radius of node i . A transmission from node i can be successfully received at node j if and only if following two conditions hold:

- (i) $\text{dist}(i, j) \leq r_i$;
- (ii) $\text{dist}(k, j) \geq (1 + \Delta)\text{dist}(i, j)$ for each node $k \neq i$ which transmits at the same time, where Δ is a protocol-specified guard-zone to prevent interference.

We further assume that at each time slot, at most W bits can be transmitted in a successful transmission.

Time scale of mobility: Two time scales of mobility are considered in this paper.

- (1) Fast mobility: The mobility of nodes is at the same time scale as the data transmission, so W is a constant independent of n and only one-hop transmissions are feasible in single time slot.
- (2) Slow mobility: The mobility of nodes is much slower than the wireless transmission, so $W = \omega(n)$. Under this assumption, the packet size can be scaled as $W/H(n)$ for $H(n) = O(n)$ to guarantee $H(n)$ -hop transmissions are feasible in single time slot.

Delay and throughput: We consider hard delay constraints in this paper. Given a delay constraint D , a packet is said to be successfully delivered if the destination obtains the packet within D time slots after it is sent out from the source.

Let $\Lambda_i[T]$ denote the number of bits successfully delivered to the destination of node i in time interval $[0, T]$. A throughput of λ per S-D pair is said to be feasible under the delay constraint D and loss probability constraint $\epsilon > 0$ if there exists n_0 such that for any $n \geq n_0$, there exists a coding, routing and scheduling algorithm with the property that each bit transmitted by a source is received at its destination with probability at least $1 - \epsilon$, and

$$\lim_{T \rightarrow \infty} \Pr \left(\frac{\Lambda_i[T]}{T} \geq \lambda, \forall i \right) = 1. \quad (1)$$

IV. MAIN RESULTS AND SOME INTUITION FOR THE I.I.D. MOBILITY MODELS

Recall that our objective is to maximize throughput in a wireless network subject to a delay constraint and a wireless interference constraint. More precisely, the constraints can be viewed as follows.

- (1) Wireless interference: Throughput is limited due to the fact that transmissions interfere with each other.
- (2) Mobility: A packet may not be delivered to its destination before the delay deadline since neither the packet's source nor any relay node may get close enough to the destination.

In this section, we present some heuristic arguments to obtain an upper bound on the maximum throughput subject to these two constraints and derive some key results. While the heuristics are far from precise derivations of the optimal delay-throughput tradeoffs, they may be useful in understanding the main results. In addition, the heuristic arguments provide the right order for the "hitting distance" (to be defined later) which plays

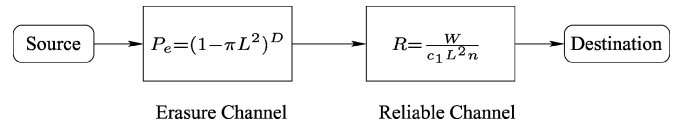


Fig. 3. Virtual-channel representation for the two-dimensional i.i.d. mobility model with fast mobiles.

a critical role in the optimal algorithms used to achieve the delay-throughput tradeoffs.

First, consider the two-dimensional i.i.d. mobility model with fast mobiles. We say that a packet *hits* its destination at time slot t if the distance between the packet and its destination is less than or equal to L . Under the two-dimensional i.i.d. mobility model, a packet hits its destination with probability πL^2 at each time slot. So, given a delay constraint D , the probability that a packet hits its destination in one of D time slots is

$$1 - (1 - \pi L^2)^D.$$

Furthermore, under the fast mobility, only one-hop transmissions are feasible at each time slot. So the transmission radius needs to be at least L to deliver packets to their destinations when their distance is L . Assume that all nodes use a common transmission radius L and that all nodes wish to transmit at each time slot, then each node has $1/(c_1 n L^2)$ fraction of time to transmit, and the throughput per S-D pair is no more than $W/(c_1 n L^2)$ where c_1 is a positive constant independent of n . Thus, the network can be regarded as a system where there are two virtual channels between each S-D pair as in Fig. 3. The packets are first sent over the erasure channel with erasure probability

$$P_e = (1 - \pi L^2)^D$$

and then over the reliable channel with rate

$$R = \frac{W}{c_1 L^2 n}$$

bits per time slot. Each source can transmit at most W bits per time slot on average. So in this virtual system, the maximum throughput of a S-D pair is

$$\begin{aligned} \lambda &= \max_L \min \left\{ W \left(1 - (1 - \pi L^2)^D \right), \frac{W}{c_1 L^2 n} \right\} \\ &= W \left(\sqrt{\frac{\pi D}{c_1 n}} \right), \end{aligned}$$

and the corresponding optimal hitting distance $L^* = b_1 / \sqrt[4]{nD}$ where $b_1 = \sqrt[4]{c_1 \pi}$.

To achieve this throughput, we first need to use the optimal L . Furthermore, a coding scheme achieving the capacity of the erasure channel is needed. Since the erasure probability is determined by L and D , which are different under different delay constraints, rate-less codes become a reasonable choice. If the encoding/decoding time is negligible compared to the delay constraint D , then any rate-less codes could be used in the algorithms presented in this paper. We will use Raptor codes because Raptor codes have linear encoding/decoding complexity, which guarantees that the encoding/decoding time will not affect our order results even if it is not negligible. We

also note that the idea of using coding is one of the keys to achieve the optimal delay–throughput tradeoffs. In some of the previous works, small delays are achieved by broadcasting each data packet to multiple relay nodes. From our virtual channel representation, such an approach is equivalent to using the repetition coding over the erasure channel, which is not optimal. We would like to mention that the idea of using coding to improve reliability of packet delivery has also been considered independently by Shah and Shakkottai in [16] for *ad hoc* sensor networks in a different context.

Our first result is as follows: *Under the two-dimensional i.i.d. mobility model with fast mobiles, the throughput per S-D pair is $\lambda = O\left(\sqrt{D/n}\right)$ given delay constraint D . When D is both $\omega(\sqrt[3]{n})$ and $o(n)$, this throughput can be achieved using a joint coding–scheduling algorithm.*

Note that the heuristic arguments leading up to the above result have many limitations. For example, it suggests that one can wait for the source to hit the destination to deliver the packet. In reality, such a scheme will not work since we deliver only one packet to the destination during each encounter between the S-D pair. Thus, other packets at the source which are not delivered may violate their delay constraints. This problem in the heuristic argument is because it assumes that we have an independent erasure channel for each packet despite the fact that the transmitting node is the same source. Despite the limitations, the heuristic argument surprisingly captures the delay–throughput tradeoff and the optimal hitting distance correctly up to the right order. In practice, the bound is achievable by exploiting the broadcast nature of the wireless channel to transmit each packet to several relay nodes and allowing relay nodes to independently attempt to deliver the packet the destination.

Next consider the two-dimensional i.i.d. mobility model with slow mobiles. Since multihop transmissions are feasible at each time slot, using a precise version of the result [1] which was obtained in [17], the maximum throughput per S-D pair under the slow mobility assumption is

$$\frac{W}{c_2 L \sqrt{n}}$$

where c_2 is a positive constant independent of n . We provide a crude version of the argument from [1] here for ease of readability. Suppose each node uses a transmission radius r and the distance between a S-D pair is L , then each bit has to travel L/r hops. The number of bit hops needed to satisfy a throughput requirement of λ bits/slot/node in T slots is $\lambda L T / r$. Due to the interference model, the number of simultaneous transmissions possible in one time slot is $W / (\tilde{c}_2 r^2)$ for some constant \tilde{c}_2 . Thus, we need

$$\frac{n \lambda L T}{r} \leq \frac{W T}{\tilde{c}_2 r^2}$$

or

$$\lambda \leq \frac{W}{\tilde{c}_2 L r n}.$$

Intuitively, since the total area is 1 and the number of nodes is n , the smallest radius of transmission that can be used while ensuring connectivity is given by $n \pi r^2 = 1$, so

$$\lambda \leq \frac{W}{\tilde{c}_2 L \sqrt{\pi n}}.$$

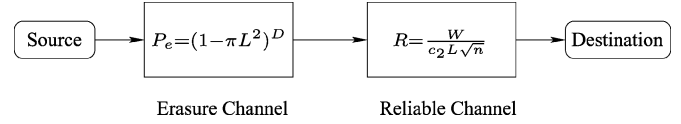


Fig. 4. Virtual-channel representation for the two-dimensional i.i.d. mobility model with slow mobiles.

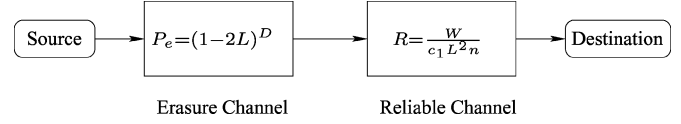


Fig. 5. Virtual-channel representation for the one-dimensional i.i.d. mobility model with fast mobiles.

That this is indeed achievable in an order sense is proved in [17], and therefore, we take λ to be $W / (c_2 L \sqrt{n})$ where $c_2 = \sqrt{\pi} \tilde{c}_2$. Then the virtual channels between an S-D pair are as depicted in Fig. 4. In this virtual system, the maximum throughput of a S-D pair is

$$\begin{aligned} \lambda &= \max_L \min \left\{ W \left(1 - (1 - \pi L^2)^D \right), \frac{W}{c_2 L \sqrt{n}} \right\} \\ &= W \left(\sqrt[3]{\frac{\pi D}{c_2^2 n}} \right) \end{aligned}$$

and the optimal hitting distance $L^* = b_2 / \sqrt[3]{n D^2}$ where $b_2 = \sqrt[3]{c_2 \pi}$. This throughput can also be achieved using a joint coding–scheduling scheme.

The main result is as follows: *Under the two-dimensional i.i.d. mobility model with slow mobiles, the throughput per S-D pair is $\lambda = O\left(\sqrt[3]{D/n}\right)$ given a delay constraint D . This throughput can be achieved using a joint coding–scheduling scheme when D is both $\omega(1)$ and $o(n)$.*

We next consider the one-dimensional i.i.d. mobility model. Note that we have assumed that the orbits of the source and the destination are parallel to each other. Thus, the probability that a packet hits its destination is at most $2L$ at each time slot, and the probability of no hitting during D consecutive time slots is

$$P_e = (1 - 2L)^D.$$

For fast mobiles, the throughput per S-D pair is $W / (c_1 L^2 n)$ with a common transmission radius L . Thus, the two virtual channels are as depicted in Fig. 5. In this virtual system, the maximum throughput λ is

$$\begin{aligned} \lambda &= \max_L \min \left\{ W \left(1 - (1 - 2L)^D \right), \frac{W}{c_1 L^2 n} \right\} \\ &= W \left(\sqrt[3]{\frac{4D^2}{c_1^2 n}} \right). \end{aligned}$$

The main result is as follows: *Under the one-dimensional i.i.d. mobility with fast mobiles, the throughput per S-D pair is $\lambda = O\left(\sqrt[3]{D^2/n}\right)$ given a delay constraint D . This throughput can be achieved using a joint coding–scheduling algorithm when D is both $\omega(\sqrt[3]{n})$ and $o(\sqrt{n} / \sqrt[3]{\log n})$.*

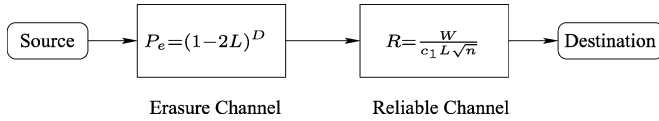


Fig. 6. Virtual-channel representation for the one-dimensional i.i.d. mobility model with slow mobiles.

For the slow mobility case, the virtual channels between an S-D pair are as shown in Fig. 6. In this virtual system, the maximum throughput λ is

$$\begin{aligned} \lambda &= \max_L \min \left\{ W \left(1 - (1 - 2L)^D \right), \frac{W}{c_2 L \sqrt{n}} \right\} \\ &= W \left(\sqrt[4]{\frac{4D^2}{c_2^2 n}} \right). \end{aligned}$$

The main result is as follows: *Under the one-dimensional i.i.d. mobility model with slow mobiles, the throughput per S-D pair is $\lambda = O\left(\sqrt[4]{D^2/n}\right)$ given a delay constraint D . This throughput can be achieved using a joint coding–scheduling algorithm when $D = o(\sqrt{n}/\log^2 n)$.*

For the hybrid random walk models, similar results can be obtained by using the same intuition. However, the delays have to be larger to allow a packet to reach its destination since a mobile can only move a certain distance from its current position at one time slot.

As stated before, the crude virtual channel representation used in this section surprisingly yields the correct results. However, they do not form the basis of the proofs in the rest of the paper. Several assumptions have been made in deriving the virtual channel representation.

- (1) The hitting events for various packets are assumed to be independent, which is difficult to ensure since the same node may act as a relay for multiple packets.
- (2) It assumes a fixed hitting distance which is not reasonable to obtain an upper bound on the throughput. An upper bound must be scheme-independent.

In view of these limitations, we use the virtual channel model to only provide some insight into the results and the hitting distance we should use in the achievable algorithms, but rigorous proofs of the main results are provided in subsequent sections.

V. TWO-DIMENSIONAL I.I.D. MOBILITY MODEL, FAST MOBILES

In this section, we investigate the two-dimensional i.i.d. mobility model with fast mobiles. Assuming that all mobiles have wireless communication and coding capability, we investigate the maximum throughput the network can achieve by using relaying and coding to recover packet loss as discussed in the heuristic arguments. Given a delay constraint D , we will first prove that the maximum throughput per S-D pair which can be supported by the network is $O\left(\sqrt{D/n}\right)$. Then a joint coding–scheduling scheme will be proposed to achieve the maximum throughput when D is both $\omega(\sqrt[3]{n})$ and $o(n)$.

A. Upper Bound

In this subsection, we show the maximum throughput the network can support without network coding, i.e., under the following assumption.

Assumption 1: Packets destined for different nodes cannot be encoded together. Further, we assume that coding is only used to recover from erasures and not for data compression. Specifically, we assume that at least k coded packets are necessary to recover k data packets, where all packets (coded or uncoded) are assumed to be of the same size.

Assumption 1 is the only significant restriction imposed on coding, routing and scheduling schemes. We also make the following assumption.

Assumption 2: A new coded packet is generated right before the packet is sent out. The node generating the coded packet does not store the packet in its buffer.

Assumption 2 is not restrictive since the information contained in the new packet is already available at the node.

Assumption 3: Once a node receives a packet (coded or uncoded), the packet is not discarded by the node till its deadline expires.

Assumption 3 is not restrictive since we are studying an upper bound on the throughput in this section.

Next we introduce the following notations, which will be used in our proof:

- b : Index of a bit stored in the network. Bit b could be either a bit of a data packet or a bit of a coded packet. If a node generates a copy of a packet to be stored in another node, then the bits in the copy are given different indices than the bits in the original packet.
- d_b : The destination of bit b .
- c_b : The node storing bit b .
- t_b : The time slot at which bit b is generated.
- S_b : If bit b is delivered to its destination, then S_b is the transmission radius used to deliver b .
- $\mathcal{R}[T]$: The set of all bits stored at relay nodes at time slot T . We do not include bits that are still in their source node in defining $\mathcal{R}[T]$.
- $\Lambda[T] : \Lambda[T] = \sum_{i=1}^n \Lambda_i[T]$.

Assume that the delay constraint is D , and a data packet is processed by the source node at time slot t_p . Then the data packet is said to be active from time slot t_p to $t_p + D - 1$. A bit b is said to be active if at least one data packet encoded into the packet containing bit b has not expired. It is easy to see that any bit expires at most D time slots after the bit is generated. Also a bit is said to be good if it is active when delivered to its destination. Now let $\tilde{\Lambda}[T]$ denote the number of good bits delivered to destinations in $[0, T]$. Without loss of generality, we assume good bits are indexed from 1 to $\tilde{\Lambda}[T]$. Note that expired bits might help decode good source bits but would not contribute to the total throughput, so we have

$$\tilde{\Lambda}[T] \geq \Lambda[T] \quad (2)$$

where $\Lambda[T]$ is the number of good source bits successfully recovered at destinations.

Next we present three fundamental constraints. In the following lemma, inequalities (3) and (4) hold since the total number of bits transmitted or received in T time slots cannot exceed nWT . Inequality (5) holds since under the protocol model, discs of radius $\Delta r_i/2$ around the receivers should be mutually disjoint from each other.

Lemma 1: For any mobility model, the following inequalities hold:

$$\tilde{\Lambda}[T] \leq nWT \quad (3)$$

$$|\mathcal{R}[T]| \leq nWT \quad (4)$$

$$\sum_{b=1}^{\tilde{\Lambda}[T]} \frac{\Delta^2}{16} (S_b)^2 \leq \frac{WT}{\pi} \quad (5)$$

where $|\mathcal{R}[T]|$ is the cardinality of the set $\mathcal{R}[T]$.

Proof: Since each node can transmit at most W bits per time slot, the total number of bits transmitted in T time slots is less than nWT which implies inequalities (3) and (4). Inequality (5) was proved in [18]. \square

We first consider the scenario where packet relaying is not allowed, i.e., packets need to be directly transmitted from sources to destinations. In the following lemma, we show that the throughput in this case is at most $\Theta(1/\sqrt{n})$ even without the delay constraint.

Lemma 2: Consider the two-dimensional i.i.d. mobility model with fast mobiles. Suppose that packets have to be directly transmitted from sources to destinations, then

$$\frac{8\sqrt{2}}{\Delta} WT\sqrt{n} \geq E[\Lambda[T]]. \quad (6)$$

Proof: First from the Cauchy–Schwarz inequality and inequality (5), we have

$$\begin{aligned} \left(\sum_{b=1}^{\tilde{\Lambda}[T]} S_b \right)^2 &\leq \left(\sum_{b=1}^{\tilde{\Lambda}[T]} 1 \right) \left(\sum_{b=1}^{\tilde{\Lambda}[T]} (S_b)^2 \right) \\ &\leq \tilde{\Lambda}[T] \frac{16WT}{\pi\Delta^2} \end{aligned}$$

which implies

$$E \left[\sum_{b=1}^{\tilde{\Lambda}[T]} S_b \right] \leq \left(\sqrt{\frac{16WT}{\pi\Delta^2}} \right) E \left[\sqrt{\tilde{\Lambda}[T]} \right]. \quad (7)$$

This gives an upper bound on the expected distance traveled. Next, we bound the total number of times that each mobile gets within a distance L of its destination for $L \in [0, 1/2]$. From the i.i.d. mobility assumption, we have that for any i, j , and t

$$\Pr(\text{dist}(i, j)(t) \leq L) = \pi L^2$$

which implies

$$E \left[\sum_{t=1}^T \left(\sum_{i=1}^n \mathbf{1}_{\text{dist}(i, ((i+1) \bmod n))(t) \leq L} \right) \right] = \pi L^2 nT.$$

Since at most W bits can be transmitted at each time slot, we further have

$$\sum_{b=1}^{\tilde{\Lambda}[T]} \mathbf{1}_{S_b \leq L} \leq W \sum_{t=1}^T \sum_{i=1}^n \mathbf{1}_{\text{dist}(i, ((i+1) \bmod n))(t) \leq L}.$$

Taking expectation on both sides of above inequality, we obtain

$$E \left[\tilde{\Lambda}[T] \right] - E \left[\sum_{b=1}^{\tilde{\Lambda}[T]} \mathbf{1}_{S_b > L} \right] \leq W\pi L^2 nT. \quad (8)$$

Now using Jensen's inequality and inequalities (7) and (8), we can conclude that

$$\begin{aligned} \sqrt{\frac{16WT}{\pi\Delta^2}} E \left[\tilde{\Lambda}[T] \right] &\geq \left(\sqrt{\frac{16WT}{\pi\Delta^2}} \right) E \left[\sqrt{\tilde{\Lambda}[T]} \right] \\ &\geq E \left[\sum_{b=1}^{\tilde{\Lambda}[T]} S_b \right] \\ &\geq LE \left[\sum_{b=1}^{\tilde{\Lambda}[T]} \mathbf{1}_{S_b > L} \right] \\ &\geq L \left(E \left[\tilde{\Lambda}[T] \right] - W\pi L^2 nT \right). \quad (9) \end{aligned}$$

Note that inequality (9) holds for any $L \in [0, 1/2]$. We choose

$$L^* = \sqrt{\frac{E[\tilde{\Lambda}[T]]}{2\pi nWT}}$$

which is less than $1/2$ since $\tilde{\Lambda}[T] \leq nWT$. Substituting L^* into inequality (9), we have

$$\sqrt{\frac{16WT}{\pi\Delta^2}} E \left[\tilde{\Lambda}[T] \right] \geq \frac{1}{2} L^* E \left[\tilde{\Lambda}[T] \right]$$

which implies that

$$\frac{8\sqrt{2}}{\Delta} WT\sqrt{n} \geq E \left[\tilde{\Lambda}[T] \right].$$

The lemma then follows from inequality (2). \square

Next we investigate the maximum throughput the network can support using coding, routing, and scheduling schemes. We have obtained an upper bound on the number of bits directly transmitted from sources to destinations in Lemma 2. To bound the maximum throughput with relaying, we will calculate the number of bits transmitted from relays to destinations in the following analysis.

Theorem 3: Consider the two-dimensional i.i.d. mobility model with fast mobiles, and assume that Assumptions 1–3 hold. Then given a delay constraint D , we have that

$$\frac{8\sqrt{2}WT}{\Delta} \sqrt{n} \left(\sqrt{D} + 1 \right) \geq E[\Lambda[T]]. \quad (10)$$

Proof: In the proof of the theorem, we treat active bits at relays and active bits at sources differently since we can bound the number of active bits at relays using inequality (4), while the number of active bits at sources could be larger. Let $\tilde{\Lambda}^r[T]$

denote the number of good bits delivered directly from relays to destinations in $[0, T]$. Without loss of generality, we assume these good bits are indexed from 1 to $\tilde{\Lambda}^r[T]$. Similar to inequality (7), we first have

$$E \left[\sum_{b=1}^{\tilde{\Lambda}^r[T]} S_b \right] \leq \left(\sqrt{\frac{16WT}{\pi\Delta^2}} \right) E \left[\sqrt{\tilde{\Lambda}^r[T]} \right]. \quad (11)$$

Let \tilde{L}_b denote the minimum distance between node d_b and node c_b from time slot t_b to time slot $t_b + D - 1$, i.e.,

$$\tilde{L}_b = \min_{t_b \leq t \leq t_b + D - 1} \text{dist}(d_b, c_b)(t).$$

Then for any $L \in [0, 1/2]$ and any bit $b \in \mathcal{R}[T]$, we have

$$\Pr(\tilde{L}_b \leq L) = 1 - (1 - \pi L^2)^D \leq \pi L^2 D$$

which implies

$$E \left[\sum_{b \in \mathcal{R}[T]} 1_{\tilde{L}_b \leq L} \right] \leq nWT\pi L^2 D.$$

Furthermore, we have

$$\sum_{b=1}^{\tilde{\Lambda}^r[T]} 1_{S_b \leq L} \leq \sum_{b \in \mathcal{R}[T]} 1_{\tilde{L}_b \leq L}$$

which implies that

$$E \left[\sum_{b=1}^{\tilde{\Lambda}^r[T]} 1_{S_b \leq L} \right] \leq nWT\pi L^2 D. \quad (12)$$

Thus, we can conclude that

$$\begin{aligned} E \left[\sum_{b=1}^{\tilde{\Lambda}^r[T]} S_b \right] &\geq E \left[\sum_{b=1}^{\tilde{\Lambda}^r[T]} S_b 1_{S_b > L} \right] \\ &\geq LE \left[\sum_{b=1}^{\tilde{\Lambda}^r[T]} 1_{S_b > L} \right] \\ &\geq L \left(E[\tilde{\Lambda}^r[T]] - E \left[\sum_{b=1}^{\tilde{\Lambda}^r[T]} 1_{S_b \leq L} \right] \right) \\ &\geq LE[\tilde{\Lambda}^r[T]] - nWT\pi L^3 D \end{aligned} \quad (13)$$

where the last inequality follows from inequality (12).

Now using Jensen's inequality and inequalities (11) and (13), we have that for any $L \in [0, 1/2]$

$$\sqrt{\frac{16WT}{\pi\Delta^2}} E \left[\tilde{\Lambda}^r[T] \right] \geq LE[\tilde{\Lambda}^r[T]] - nWT\pi L^3 D. \quad (14)$$

Substituting

$$L^* = \sqrt{\frac{E[\tilde{\Lambda}^r[T]]}{2nWT\pi D}}$$

into inequality (14), we can conclude that

$$\frac{8\sqrt{2}WT}{\Delta} \sqrt{nD} \geq E \left[\tilde{\Lambda}^r[T] \right]. \quad (15)$$

The theorem follows from inequalities (6), (15), and (2). \square

From Theorem 3, we can conclude that the throughput per S-D is $O(\sqrt{D/n})$ given a delay constraint D .

B. Joint Coding–Scheduling Algorithm

In Section IV, we motivated the need to first encode data packets. In this subsection, we use Raptor codes and propose a joint coding–scheduling scheme to achieve the maximum throughput obtained in Theorem 3.

Motivated by the heuristic argument in Section IV, we divide the unit square into square cells with each side of length equal to $1/\sqrt[4]{nD}$, which is of the same order as the optimal hitting distance. In our scheme, we will allow final delivery of a packet to its destination only when a relay carrying the packet is in the same cell as the destination. Thus, a packet is delivered only when the relay and destination are within a distance of $\sqrt{2}/\sqrt[4]{nD}$, which is also the same as the hitting distance calculated in Section IV except for a constant factor which does not play a role in the order calculations. The mean number of nodes in each cell will be denoted by M and is equal to $\sqrt{n/D}$. The transmission radius of each node is chosen to be $\sqrt{2}/\sqrt[4]{nD}$ so that any two nodes within a cell can communicate with each other. This means that, given the interference constraint, two nodes in a cell can communicate if all nodes in cells within a fixed distance from the given cell stay silent. Each time slot is further divided into C minislots and each cell is guaranteed to be active in at least one minislot within each time slot. Assume $C = 9$. The reason we use nine minislots is that if a node in a cell is active, then no other nodes in any of its neighboring eight cells can be active, but nodes outside this neighborhood can be active. Further, we denote the packet size to be $W/(2C)$ so that two packets can be transmitted in each minislot.

A cell is said to be a *good cell* at time t if the number of nodes in the cell is between $9M/10 + 1$ and $11M/10$. We also define and categorize packets into four different types.

- Data packets: Uncoded data packets that have to be transmitted by the sources and received by the destinations.
- Coded packets: Packets generated by Raptor codes. We let (i, k) denote the k th coded packet of node i .
- Duplicate packets: Each coded packet could be broadcast to other nodes to generate multiple copies, called duplicate packets. We let (i, k, j) denote a copy of (i, k) carried by node j , and (i, k, J) to denote the set of all copies of coded packet (i, k) .
- Deliverable packets: Duplicate packets that happen to be in the same cell as L from their destinations.

We now describe our coding/scheduling algorithm, which guarantees a throughput of $\frac{9W}{500C} \sqrt{\frac{D}{n}}$ per S-D pair given a delay constraint $6D$. Here, we consider a delay constraint $6D$ instead of D to simplify notations, which obviously does not affect our

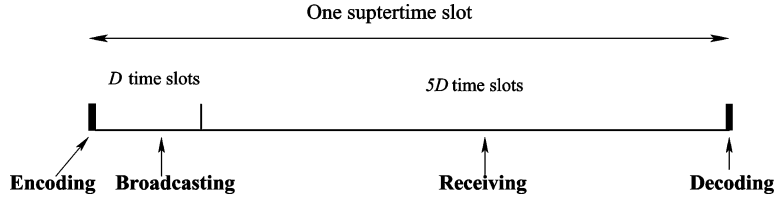


Fig. 7. Time structure of Algorithm I.

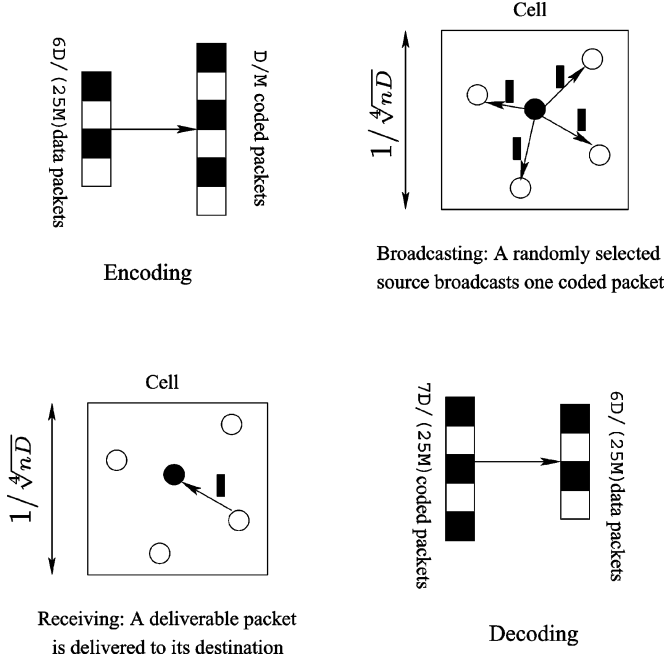


Fig. 8. Outline of Algorithm I.

order results. The time structure and procedure of Algorithm I are illustrated in Figs. 7 and 8.

Joint Coding-Scheduling Algorithm I: We group every $6D$ time slots into a super-time slot. At each super-time slot, the nodes transmit packets as follows.

- (1) **Raptor encoding:** Each source takes $6D/(25M)$ data packets, and uses Raptor codes to generate D/M coded packets.
- (2) **Broadcasting:** This step consists of D time slots. At each time slot, the nodes execute the following tasks:
 - (i) In each good cell, one node is randomly selected. If the selected node has not already transmitted all of its D/M coded packets, then it broadcasts a coded packet that was not previously transmitted to $9M/10$ other nodes in the cell during the minislot allocated to that cell. Recall that our choice of packet size allows one node in every good cell to transmit during every time slot.
 - (ii) All nodes check the duplicate packets they have. If more than one duplicate packets have the same destination, select one at random to keep and drop the others.
- (3) **Receiving:** This step consists of $5D$ time slots. At each time slot, if a cell contains no more than two deliver-

able packets, the deliverable packets are delivered to their destinations using one-hop transmissions during the minislot allocated to that cell; otherwise, no node in the cell attempts to transmit. At the end of this step, all undelivered packets are dropped. The destinations decode the received coded packets using Raptor decoding.

Note that in describing the algorithm, we did not account for the delays in Raptor encoding and decoding. However, Raptor codes have linear encoding and decoding complexity. Hence, even if these delays are taken into account, our order results will not change.

Theorem 4: Consider Joint Coding-Scheduling Algorithm I. Suppose D is both $\omega(\sqrt[3]{n})$ and $o(n)$, and the delay constraint is $6D$. Then given any $\epsilon > 0$, there exists n_0 such that for any $n \geq n_0$, every data packet sent out can be recovered at the destination with probability at least $1 - \epsilon$, and furthermore

$$\lim_{T \rightarrow \infty} \Pr \left(\frac{\Lambda_i[T]}{T} \geq \frac{9W}{500C} \sqrt{\frac{D}{n}} \forall i \right) = 1. \quad (16)$$

Proof: Let t_s denote the t_s th super-time slot. In Appendix D, we show that the following events ((17)–(19)) happen with high probability.

Broadcasting: At least $16D/(25M)$ coded packets from a source are successfully duplicated after the broadcasting step with high probability, where a coded packet is said to be successfully duplicated if the packet is in at least $4M/5$ distinct relay nodes. Letting $A_i[t_s]$ denote the number of coded packets of source i which are successfully duplicated in super-time slot t_s , we first have that there exists n_1 such that for any $n \geq n_1$

$$\Pr \left(A_i[t_s] \geq \frac{16D}{25M} \right) \geq 1 - 3e^{-\frac{D}{500M}}. \quad (17)$$

Receiving: At least $7D/(25M)$ distinct coded packets from a source are delivered to its destination after the receiving step with high probability. Letting $B_i[t_s]$ denote the number of distinct coded packets delivered to destination $i + 1$ in super-time slot t_s , we have that there exists n_2 such that for all $n \geq n_2$

$$\Pr \left(B_i[t_s] \geq \frac{7D}{25M} \mid A_i[t_s] \geq \frac{16D}{25M} \right) \geq 1 - 2e^{-\frac{D}{180M}}. \quad (18)$$

Decoding: The $6D/25M$ data packets from a source are recovered with high probability. Letting $\mathcal{E}_i[t_s]$ denote the event such that all $6D/(25M)$ data packets are fully recovered by destination $i + 1$, we have that

$$\Pr \left(\mathcal{E}_i[t_s] \mid B_i[t_s] \geq \frac{7D}{25M} \right) \geq 1 - \left(\frac{M}{D} \right)^a \quad (19)$$

for some $a > 0$.

Recall that $M = \sqrt{n/D}$ and $D = \omega(\sqrt[3]{n})$, so

$$\lim_{n \rightarrow \infty} \frac{M}{D} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{D\sqrt{D}} = 0.$$

Combining inequalities (17)–(19), we can conclude that for any $\epsilon \leq 1/19$, there exists $n_0 \geq \max\{n_1, n_2\}$ such that for $n \geq n_0$

$$\Pr(\mathcal{E}_i[t_s]) \geq 1 - \epsilon \quad (20)$$

which implies that every data packet sent out can be recovered with probability at least $1 - \epsilon$. Since $1 - \epsilon \geq 18/19$, from the Chernoff bound (see Lemma 20 provided in Appendix C for convenience), we can conclude that for $n \geq n_0$

$$\Pr\left(\sum_{t_s=1}^{T_s} 1_{\mathcal{E}_i[t_s]} \geq \frac{9}{10}T_s\right) \geq 1 - e^{-\frac{T_s}{800}}$$

where we choose $\delta = 1/20$ in Lemma 21. Note that

$$\sum_{t_s=1}^{T_s} 1_{\mathcal{E}_i[t_s]} \geq \frac{9}{10}T_s$$

implies at least

$$\frac{9}{10}T_s \times \frac{6D}{25M} \times \frac{W}{2C} = \frac{27W}{250C} \frac{DT_s}{M} = \frac{27W}{250C} DT_s \sqrt{\frac{D}{n}}$$

bits are successfully transmitted from node i to node $i+1$ in T_s super-time slots. Since each super-time slot consists of $6D$ time slots, we can conclude that for $n \geq n_0$

$$\Pr\left(\Lambda_i[6DT_s] \geq \frac{27W}{250C} DT_s \sqrt{\frac{D}{n}} \forall i\right) \geq 1 - ne^{-\frac{T_s}{800}}$$

which implies that, for a fixed $n \geq n_0$

$$\lim_{T \rightarrow \infty} \Pr\left(\frac{\Lambda_i[T]}{T} \geq \frac{9W}{500C} \sqrt{\frac{D}{n}} \forall i\right) = 1. \quad \square$$

VI. TWO-DIMENSIONAL I.I.D. MOBILITY MODEL, SLOW MOBILES

In this section, we investigate the two-dimensional i.i.d. mobility model with slow mobiles. Given a delay constraint D , we first prove the maximum throughput per S-D pair which can be supported by the network is $O(\sqrt[3]{D/n})$. Then a joint coding–scheduling scheme is proposed to achieve the maximum throughput.

A. Upper Bound

Let \hat{t}_b denote the time slot in which bit b is delivered to its destination. Under the slow mobility, the delivery in \hat{t}_b could use multihop transmissions, so we further define following notations.

- H_b : The number hops bit b travels in time slot \hat{t}_b .
- L_b : The Euclidean distance bit b travels in time slot \hat{t}_b .

- S_b^h : The transmission radius used in hop h for $1 \leq h \leq H_b$. Similar to Lemma 1, we have following results.

Lemma 5: For any mobility model, the following inequalities hold

$$\sum_{b=1}^{\tilde{\Lambda}[T]} \sum_{h=1}^{H_b} 1 \leq nWT \quad (21)$$

$$\sum_{b=1}^{\tilde{\Lambda}[T]} \sum_{h=1}^{H_b} \frac{\Delta^2}{16} (S_b^h)^2 \leq \frac{WT}{\pi}. \quad (22)$$

□

Similar to the fast mobility cases, we first consider the throughput under the assumption that the packets can only be delivered to destinations from sources.

Lemma 6: Consider the two-dimensional i.i.d. mobility model with slow mobiles. Suppose that packets have to be directly transmitted to destinations from sources, then

$$\frac{4\sqrt[3]{2}WT}{3^{3/2}\Delta} \sqrt[3]{n} \geq E[\Lambda[T]]. \quad (23)$$

Proof: First from the Cauchy–Schwartz inequality and Lemma 5, we have that

$$\begin{aligned} \left(\sum_{b=1}^{\tilde{\Lambda}[T]} \sum_{h=1}^{H_b} S_b^h\right)^2 &\leq \left(\sum_{b=1}^{\tilde{\Lambda}[T]} \sum_{h=1}^{H_b} 1\right) \left(\sum_{b=1}^{\tilde{\Lambda}[T]} \sum_{h=1}^{H_b} (S_b^h)^2\right) \\ &\leq nWT \sum_{b=1}^{\tilde{\Lambda}[T]} \sum_{h=1}^{H_b} (S_b^h)^2 \\ &\leq \frac{16W^2T^2n}{\pi\Delta^2} \end{aligned}$$

which implies

$$\frac{4WT\sqrt{n}}{\Delta\sqrt{\pi}} \geq \sum_{b=1}^{\tilde{\Lambda}[T]} L_b \quad (24)$$

since $\sum_{h=1}^{H_b} S_{(i,b)}^h \geq L_b$. The rest of the proof is same as the proof of Lemma 2. □

Theorem 7: Consider the two-dimensional i.i.d. mobility model with slow mobiles, and assume that Assumptions 1–3 hold. Then given a delay constraint D , we have

$$\frac{4\sqrt[3]{2}WT}{3^{3/2}\Delta} \sqrt[3]{n} \left(\sqrt[3]{D} + 1\right) \geq E[\Lambda[T]]. \quad (25)$$

Proof: Based on inequality (24) and the argument of Theorem 3. □

From Theorem 7, we can conclude that the throughput per S-D is $O(\sqrt[3]{D/n})$ given a delay constraint D .

B. Joint Coding–Scheduling Algorithm

In this subsection, we propose a joint coding–scheduling scheme to achieve the throughput suggested in Theorem 7. In

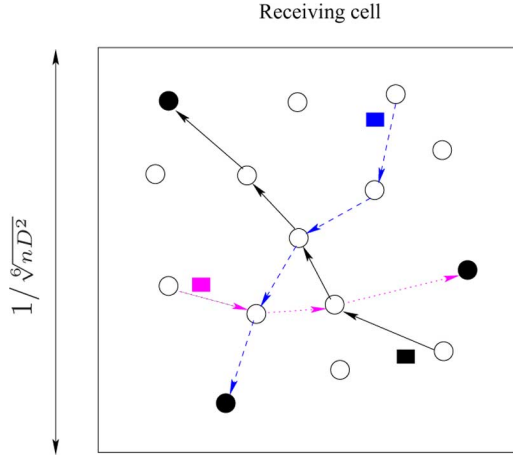


Fig. 9. Multihop delivery in the receiving step of Algorithm II.

the receiving step, we divide the unit square into square cells with each side of length equal to $1/\sqrt[6]{nD^2}$, which is of the same order as the optimal hitting distance obtained in Section IV. The mean number of nodes in each cell will be denoted by M_2 and is equal to $\sqrt[3]{n/D}$. The packet size is chosen to be

$$\frac{10W}{11c_s C \sqrt{M_2}}$$

so that at each time slot, all nodes in a good cell can transmit one packet to some other node in the same cell by using the highway algorithm proposed in [17] (see Appendix B), where c_s is a constant independent of n , and multiple packets can be delivered as illustrated in Fig. 9. In the broadcasting step, the unit square is divided into square cells with each side of length equal to $1/\sqrt[6]{n^2 D}$. The mean number of nodes in each will be denoted by M_1 and is equal to $\sqrt[3]{n/D}$. In the broadcasting step, the transmission radius of each nodes is chosen to be $\sqrt{2}\sqrt[6]{n^2 D}$. Note the packet size is

$$\frac{10W}{11c_s C \sqrt{M_2}} = \frac{10W}{11c_s C M_1}.$$

So in the broadcasting step, all nodes in a good cell could be scheduled to broadcast one coded packet at one minislot. Also note that $M_1 M_2 D/n = 1$.

Joint Coding–Scheduling Algorithm II: We group every $16D$ time slots into a super-time slot. At each super-time slot, the nodes transmit packets as follows.

- (1) **Raptor encoding:** Each source takes $2D/5$ data packets, and uses Raptor codes to generate D coded packets.
- (2) **Broadcasting:** The unit square is divided into a regular lattice with n/M_1 cells. This step consists of D time slots. At each time slot, the nodes execute the following tasks.
 - (i) In each good cell, the nodes take their turns to broadcast a coded packet to $9M_1/10$ other nodes in the cell. We use the same definition of a good cell as in Algorithm I, i.e., the number

of nodes in a good cell should be with a factor of the mean, where the factor is required to lie in the interval $[0.9, 1.1]$.

- (ii) All nodes check the duplicate packets they have. If more than one duplicate packet is destined to the same destination, randomly keep one and drop the others.
- (3) **Receiving:** The unit square is divided into a regular lattice with n/M_2 cells. This step consists of $15D$ time slots. At each time slot, the nodes in a good cell execute the following tasks in the minislot allocated to that cell.
 - (i) Each node containing deliverable packets randomly selects a deliverable packet, and sends a request to the corresponding destination.
 - (ii) Each destination only accepts one request and refuses the others.
 - (iii) The nodes whose requests are accepted transmit the deliverable packets to their destinations using the highway algorithm proposed in [17].

At the end of this step, all undelivered packets are dropped. Destinations use Raptor decoding to obtain the source packets. Note that one requires some overhead in obtaining route to the destination to perform Step (3) (i) above. As in previous works, we assume that this overhead is small since one can transmit many packets in each time slot, under the slow mobility assumption.

Theorem 8: Consider Joint Coding–Scheduling Algorithm II. Suppose D is both $\omega(1)$ and $o(n)$, and the delay constraint is $16D$. Then given any ϵ , there exists n_0 such that for any $n \geq n_0$, every data packet sent out can be recovered at the destination with probability at least $1 - \epsilon$, and furthermore

$$\lim_{T \rightarrow \infty} \Pr \left(\frac{A_i[T]}{T} \geq \left(\frac{9W}{440c_s C} \right) \sqrt[3]{\frac{D}{n}} \forall i \right) = 1. \quad (26)$$

Proof: Similar to (17), we can first obtain the following result.

Broadcasting: At least $4D/5$ coded packets from a source are successfully duplicated after the broadcasting step with high probability, i.e.,

$$\Pr \left(A_i[t_s] \geq \frac{4}{5}D \right) \geq 1 - 3e^{-\frac{D}{100000}} \quad (27)$$

where a coded packet is said to be successfully duplicated if the packet is in $4M_1/5$ distinct relay nodes.

In Appendix E, we also show that the following event happens with high probability.

Receiving: At least $D/2$ distinct coded packets from a source are delivered to its destination after the receiving step with high probability, i.e.,

$$\Pr \left(B_i[t_s] \geq \frac{D}{2} \mid A_i[t_s] \geq \frac{4}{5}D \right) \geq 1 - e^{-\frac{D}{5000}}. \quad (28)$$

Based on inequalities (27) and (28), the theorem can be proved by following the argument in Theorem 4. \square

VII. TWO-DIMENSIONAL HYBRID RANDOM WALK MODEL, FAST MOBILES

In this section, we study the two-dimensional hybrid random walk model with fast mobiles. We will obtain the maximum throughput for $D = \omega(|\log S|/S^2)$, and then show that the maximum throughput can be achieved using Joint Coding–Scheduling Algorithm I under some additional constraints on D .

A. Upper Bound

Theorem 9: Consider the two-dimensional hybrid random walk model with fast mobiles. Assume that step-size $S = o(1)$ and Assumptions 1–3 hold. Then given delay constraint $D = \omega(|\log S|/S^2)$, we have

$$\frac{48\sqrt{2}WT}{\Delta\sqrt{\pi}}\sqrt{n}(\sqrt{D}+1) \geq E[\Lambda[T]]. \quad (29)$$

Proof: For any $L \in [0, S/\sqrt{\pi}]$, it is shown in Appendix G that

$$\Pr(\tilde{L}_b \leq L) \leq 36L^2D. \quad (30)$$

Inequality (29) then follows from the proof of Theorem 3. \square

B. Joint Coding–Scheduling Algorithm

From Theorem 9, we can see that the optimal delay–throughput tradeoffs of the two-dimensional hybrid random walk model with fast mobiles is similar to the one of the two-dimensional i.i.d. mobility model. It motivates us to consider Joint Coding–Scheduling Algorithm I. We will show that the optimal tradeoff can be achieved using Joint Coding–Scheduling Algorithm I with the following modifications: $2D/(25M)$ data packets are coded into D/M coded packets.

Theorem 10: Consider the two-dimensional hybrid random walk model with fast mobiles. Suppose that S is $o(1)$, D is both $\omega(\max\{\log^2 n | \log S|/(S^6), \sqrt[3]{n} \log n\})$ and $o(n/(\log^2 n))$, and the delay constraint is $6D$. Then given any ϵ there exists n_0 such that for any $n \geq n_0$, every data packet sent out can be recovered at the destination with probability at least $1 - \epsilon$, and

$$\lim_{T \rightarrow \infty} \Pr\left(\frac{\Lambda_i[T]}{T} \geq \left(\frac{W}{2520}\right) \left(\sqrt{\frac{D}{n}}\right) \forall i\right) = 1.$$

by using the modified Joint Coding–Scheduling Algorithm I.

Proof: We consider one super-time slot which consists of $6D$ time slots, and calculate the probability that the $2D/(25M)$ data packets from node i are fully recovered at the destination, where $M = \sqrt{n/D}$ is the mean number of nodes in each cell. In Appendix H, we show that the following events happen with high probability.

Node distribution: All cells are good during the entire super-time slot with high probability. Letting \mathcal{G} denote this event, we have

$$\Pr(\mathcal{G}) \geq 1 - \frac{1}{n^2}. \quad (31)$$

Broadcasting: At least $16D/(25M)$ coded packets from a source are successfully duplicated after the broadcasting step with high probability, i.e.,

$$\Pr\left(A_i \geq \frac{16D}{25M} \middle| \mathcal{G}\right) \geq 1 - \frac{55D}{n} - e^{-\frac{D}{600M}} \quad (32)$$

where a coded packet is said to be successfully duplicated if the packet is in at least $4M/5$ distinct relay nodes.

Receiving: At least $3D/(25M)$ distinct coded packets from a source are delivered to its destination after the receiving step with high probability, i.e.,

$$\Pr\left(B_i \geq \frac{3}{25} \frac{D}{M} \middle| A_i \geq \frac{16}{25} \frac{D}{M}\right) \geq 1 - 2e^{-\frac{\log D}{8500}} - e^{-\frac{D}{500M \log D}}. \quad (33)$$

From inequalities (31), (32), and (33), we can conclude that under the modified Joint Coding–Scheduling Algorithm I, at each super-time slot, the $2D/(25M)$ data packets can be successfully recovered with probability at least

$$1 - \frac{1}{n^2} - \frac{55D}{n} - e^{-\frac{D}{600M}} - 2e^{-\frac{\log D}{8500}} - e^{-\frac{D}{500M \log D}}.$$

The rest of the proof follows from the proof of Theorem 4. \square

VIII. TWO-DIMENSIONAL HYBRID RANDOM WALK MODEL, SLOW MOBILES

In this section, we study the two-dimensional hybrid random walk model with slow mobiles. We will obtain the maximum throughput for $D = \omega(|\log S|/S^2)$, and then show that the maximum throughput can be achieved using Joint Coding–Scheduling Algorithm II under some additional constraints on D .

A. Upper Bound

Theorem 11: Consider the two-dimensional hybrid random walk model with slow mobiles. Assume that step size $S = o(1)$ and Assumptions 1–3 hold. Then given delay constraint $D = \omega(|\log S|/S^2)$, we have

$$\frac{8\sqrt[3]{9}WT}{\sqrt[3/2]{\Delta\pi^2}}\sqrt[3/2]{n}(\sqrt[3]{D}+1) \geq E[\Lambda[T]]. \quad (34)$$

Proof: Follow inequality (30) and the proof of Theorem 7. \square

B. Joint Coding–Scheduling Algorithm

We will show that the optimal tradeoff can be achieved using Joint Coding–Scheduling Algorithm II with the following modification: $D/7$ data packets are coded to D coded packets.

Theorem 12: Consider the two-dimensional hybrid random walk models with slow mobiles. Suppose that S is $o(1)$ and D is both $\omega(\log^2 n | \log S|/(S^6))$ and $o(n/(\log^3 n))$, and the delay constraints is $16D$. Then under the slow mobility model, given any ϵ there exists \tilde{n}_0 such that for any $n \geq \tilde{n}_0$, every data packet

sent out can be recovered at the destination with probability at least $1 - \epsilon$, and

$$\lim_{T \rightarrow \infty} \Pr \left(\frac{\Lambda_i[T]}{T} \geq \left(\frac{W}{224\sqrt{2}c_s C} \right) \left(\sqrt[3]{\frac{D}{n}} \right) \forall i \right) = 1 \quad (35)$$

by using the modified Joint Coding–Scheduling Algorithm II.

Proof: We consider one super-time slot which consists of $16D$ time slots, and calculate the probability that the $D/7$ data packets from node i are fully recovered at the destination. Similar to Theorem 10, it is easy to verify that the following events happen with high probability.

Node distribution: All cells are good during the entire super-time slot with high probability, i.e.,

$$\Pr(\mathcal{G}) \geq 1 - \frac{1}{n^2}.$$

Broadcasting: At least $4D/5$ coded packets from a source are successfully duplicated after the broadcasting step with high probability. Specifically, we have

$$\Pr \left(A_i \geq \frac{4}{5}D \middle| \mathcal{G} \right) \geq 1 - \frac{50}{M_1^2} \quad (36)$$

where a coded packet is said to be successfully duplicated if it is in $4M_1/5$ distinct relay nodes.

Receiving: At least $D/6$ distinct coded packets from a source are delivered to its destination after the receiving step with high probability. Specifically, we have

$$\Pr \left(B_i \geq \frac{D}{6} \middle| A_i \geq \frac{4}{5}D \right) \geq 1 - 2e^{-\frac{\log D}{800}}. \quad (37)$$

From inequalities (31), (36), and (37) we can conclude that under the modified Joint Coding–Scheduling Algorithm II, at each super-time slot, the $D/7$ data packets can be successfully recovered with probability at least

$$1 - \frac{1}{n^2} - \frac{50}{M_1^2} - 2e^{-\frac{\log D}{800}}$$

and the theorem holds. \square

IX. ONE-DIMENSIONAL I.I.D. MOBILITY MODEL, FAST MOBILES

In this section, we study one-dimensional i.i.d. mobility model with fast mobiles.

A. Upper Bound

Theorem 13: Consider the one-dimensional i.i.d. mobility model with fast mobiles. Assume that Assumptions 1–3 hold, then

$$8WT \sqrt[3]{\frac{2}{\pi\Delta^2}} \sqrt[3]{n} (\sqrt[3]{D} + 1) \geq E[\Lambda[T]].$$

Proof: Recall that \tilde{L}_b is the minimum distance between node d_b and node c_b from time slot t_b to $t_b + D - 1$. If the orbits of node c_b and d_b are vertical to each other, then $\tilde{L}_b \leq L$ holds only if at some time slot t , node c_b and d_b are in the square with side length $2L$ as in Fig. 10. In this case, we have

$$\Pr(\tilde{L}_b \leq L) \leq 1 - (1 - 4L^2)^D.$$

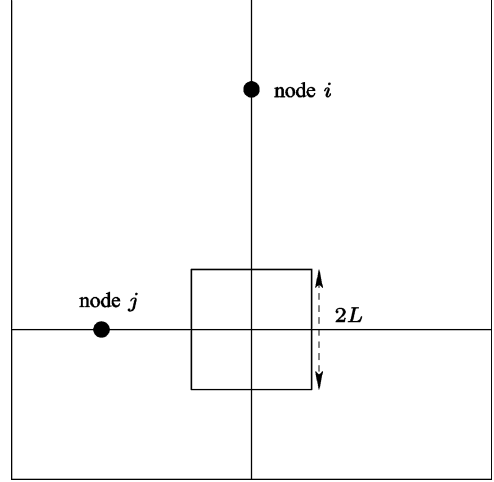


Fig. 10. Two vertical orbits.

If the orbits of node c_b and d_b are parallel to each other, then it is easy to verify that

$$\Pr(\tilde{L}_b \leq L) \leq 1 - (1 - 2L)^D.$$

Thus, for $L \leq 1/2$, we can conclude that

$$\Pr(\tilde{L}_b \leq L) \leq 1 - (1 - 2L)^D \leq 2LD. \quad (38)$$

The rest of the proof follows from the proof of Theorem 3. \square

B. Joint Coding–Scheduling Algorithm

Choose

$$M = \sqrt[3]{\frac{n}{D^2}}.$$

We divide the unit square into $\sqrt{n/M}$ horizontal rectangles, named as H-rectangles; and $\sqrt{n/M}$ vertical rectangles, named as V-rectangles as in Fig. 11. A packet is said to be destined to a rectangle if the orbit of its destination is contained in the rectangle.

The algorithms for the one-dimensional i.i.d. mobility model have four steps. The first step is the Raptor encoding. The second step is the broadcasting. In this step, the H(V)-nodes broadcast coded packets to V(H)-nodes. The third step is the transporting, where the V(H)-nodes transport the H(V)-packets to the H(V)-rectangles containing the orbits of corresponding destinations, and then broadcast packets to the H(V)-nodes whose orbits are contained in the rectangles. After the third step, all duplicate packets are carried by the nodes that move parallel with the destinations and their orbit distance is less than $\sqrt{M/n}$. The fourth step is the receiving, where the packets are delivered to the destinations. The broadcasting and transporting steps are as shown in Fig. 11.

Since duplicate copies are generated in both the broadcasting and the transporting, to distinguish them, we name the duplicate packets generated at the broadcasting as B-duplicate packets, and the duplicate packets generated at the transporting as T-duplicate packets. Also, we say a B-duplicate packet is transport-

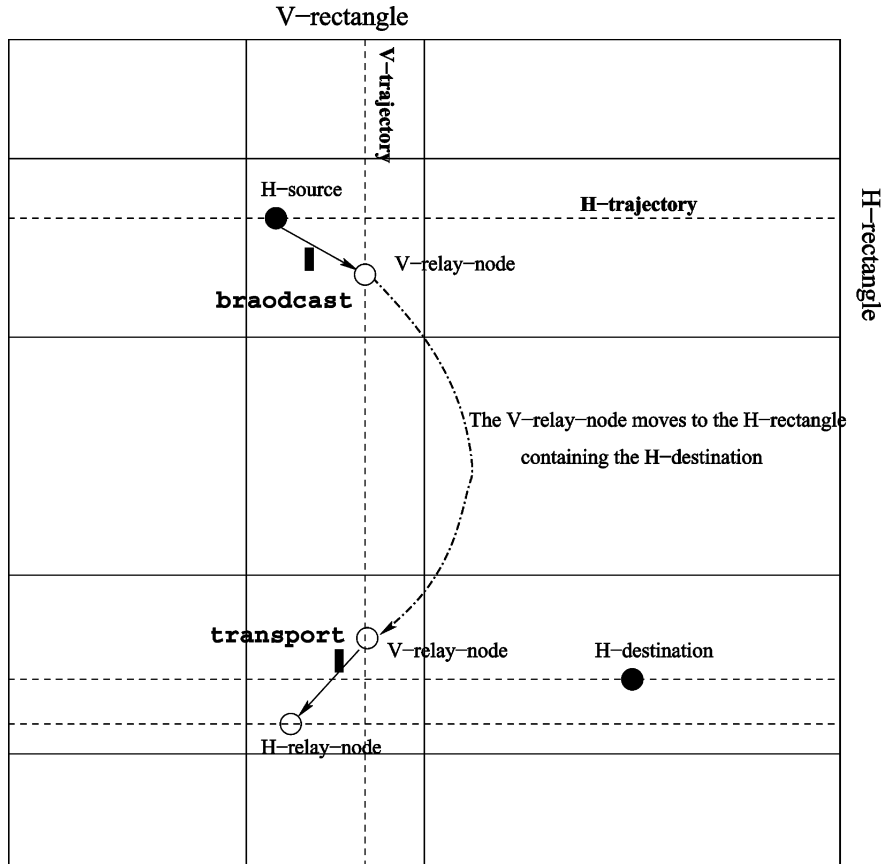


Fig. 11. Broadcasting and transporting in Algorithm III.

able if it is in the rectangle containing the orbit of the destination of the packet.

Consider a cell with area \tilde{A} and let $\tilde{M}^{H(V)}[t]$ denote the number of H(V)-nodes in the cell. For the one-dimensional mobility model, a cell is said to be a *good cell* at time slot t if

$$\frac{9}{10}\tilde{A}n + 1 \leq \tilde{M}^{H(V)}[t] \leq \frac{11}{10}\tilde{A}n.$$

Next we present the Joint Coding–Scheduling Algorithm III, which achieves the maximum throughput obtain in Theorem 13.

Joint Coding–Scheduling Algorithm III: The unit square is divided into a regular lattice with n/M cells, and the packet size is chosen to be $W/(2C)$. We group every $7D$ time slots into a super-time slot. At each super-time slot, the nodes transmit packets as follows.

- (1) **Raptor encoding:** Each source takes $2D/(35M)$ data packets, and uses the Raptor codes to generate D/M coded packets.
- (2) **Broadcasting:** This step consists of D time slots. At each time slot, the nodes execute the following tasks.
 - (i) In each good cell, one H-node and one V-node are randomly selected. If the selected H(V)-node has never been in the current cell before and not already transmitted all of its D/M coded packets, then it broadcasts a coded packet that was not previous transmitted to $9M/10$ V(H)-nodes in the cell during the minislot allocated to that cell.

- (ii) All nodes check the duplicate packets they have. If more than one B-duplicate packets are destined to the same rectangle, randomly keep one and drop the others.

- (3) **Transporting:** This step consists of D time slots. At each time slot, the nodes do the following.

- (i) Suppose that node j is a V-node, and carries B-duplicate packet (i, k, j) . Node j broadcasts (i, k, j) to $9M/10$ H-nodes in the same cell if following conditions hold: a) Node j is in a good cell; b) B-duplicate packet (i, k, j) is the only transportable H-packet in the cell.
- (ii) Each node checks the T-duplicate packets it carries. If more than one T-duplicate packet has the same destination, randomly keep one and drop the others.

- (4) **Receiving:** This step consists of $5D$ time slots. At each time slot, if a cell contains no more than two deliverable packets, the deliverable packets are delivered to their destinations using one-hop transmissions during the minislot allocated to that cell; otherwise, no node in the cell attempts to transmit. At the end of this step, all undelivered packets are dropped. The destinations decode the received coded packets using Raptor decoding.

Theorem 14: Consider Joint Coding–Scheduling Algorithm III. Suppose D is $o(\sqrt{n}/^{3/2}\sqrt{\log n})$ and $\omega(\sqrt[5]{n})$, and the delay constraint is $7D$. Then given any $\epsilon > 0$, there exists n_0 such that for any $n \geq n_0$, every data packet sent out can be

recovered at the destination with probability at least $1 - \epsilon$, and furthermore

$$\lim_{T \rightarrow \infty} \Pr \left(\frac{\Lambda_i[T]}{T} \geq \left(\frac{W}{980C} \right) \sqrt[3]{\frac{D^2}{n}} \forall i \right) = 1.$$

Proof: Consider one super-time slot and let \mathcal{G} denote the event that all cells are good in the super-time slot. The proof is based on the fact that the following events happen with high probability.

Node distribution: All cells are good during the entire super-time slot with high probability. Specifically, it is easy to verify that

$$\Pr(\mathcal{G}) \geq 1 - \frac{1}{n^2}. \quad (39)$$

Broadcasting: At least $2D/(3M)$ coded packets from a source are successfully duplicated after the broadcasting with high probability, where a coded packet is said to be successfully duplicated if it has at least $4M/5$ B-duplicate packets. Specifically, we have

$$\Pr \left(A_i \geq \frac{2D}{3M} \middle| \mathcal{G} \right) \geq 1 - \frac{40}{M}. \quad (40)$$

Transporting: At least $9D/(70M)$ coded packets from a source are successfully transported after the transporting with high probability, where a coded packet is said to be successfully transported if it has at least $4M/5$ T-duplicate copies. Letting C_i denote the number of successfully transported packets from node i , we have

$$\Pr \left(C_i \geq \frac{9D}{70M} \middle| \mathcal{G}, A_i \geq \frac{2D}{3M} \right) \geq 1 - 3e^{-\frac{D}{1500M}} - \frac{100}{M^2}. \quad (41)$$

Receiving: At least $9D/(140M)$ distinct coded packets from a source are delivered to its destination after the receiving. Specifically, we have

$$\Pr \left(B_i \geq \frac{9D}{140M} \middle| C_i \geq \frac{9D}{70M} \right) \geq 1 - 2e^{-\frac{D}{8000M}}. \quad (42)$$

Inequality (39) is easy to show, the proof of inequality (42) is similar to the proof of inequality (18), and inequalities (40) and (41) are proved in Appendix A.

From the facts above, we can conclude that the probability that the $2D/(35M)$ data packets are fully recovered in one super-time slot is at least

$$1 - \frac{1}{n^2} - \frac{40}{M} - 3e^{-\frac{D}{1500M}} - \frac{100}{M^2} - 2e^{-\frac{D}{8000M}}$$

and the theorem holds. \square

X. ONE-DIMENSIONAL I.I.D. MOBILITY MODEL, SLOW MOBILES

A. Upper Bound

Theorem 15: Consider the one-dimensional i.i.d. mobility model with slow mobiles. Assume that Assumptions 1–3 hold, then

$$4WT \sqrt[4]{\frac{4}{\pi\Delta^2}} \sqrt[3]{4n} (\sqrt{D} + 1) \geq E[\Lambda[T]].$$

Proof: Follow from inequality (38) and the proof of Theorem 7. \square

B. Joint Coding–Scheduling Algorithm

In this subsection, we propose an algorithm which achieves the delay–throughput tradeoff obtained in Theorem 15. First choose

$$M_1 = \sqrt[4]{\frac{n}{4D^2}}$$

$$M_2 = M_1^2$$

and scale the packet size to be

$$\frac{W}{4c_s M_1}.$$

Further, we divide the unit square into $\sqrt{n/M_2}$ horizontal rectangles, named as H-rectangles; and $\sqrt{n/M_2}$ vertical rectangles, named V-rectangles.

Joint Coding–Scheduling Algorithm IV: We group every $14D$ time slots into a super-time slot. At each super-time slot, the packets are coded and transmitted as follows.

- (1) **Raptor Encoding:** Each source takes $D/50$ data packets, and uses the Raptor codes to generate D coded packets.
- (2) **Broadcasting:** The unit square is divided into a regular lattice with n/M_1 cells. This step consists of D time slots. At each time slot, the nodes execute the following tasks.
 - (i) The nodes in good cells take their turns to broadcast. If node i is an H(V)-node and has never been in the current cell before, it randomly selects $9M_1/10$ V(H)-nodes and broadcasts a coded packet to them.
 - (ii) Each node checks B-duplicate packets it carries. If there are multiple B-duplicate packets destined to a same rectangle, randomly pick one and drop the others.
- (3) **Transporting:** The unit square is divided into a regular lattice with n/M_1 cells. This step consists of $2D$ time slots. At each time slot, the nodes do the following.
 - (i) Suppose node j carries duplicate packet (i, k, j) , which is an H-packet. If node j is in a good cell and (i, k, j) is transportable, node j broadcasts the packet to $9M_1/10$ H-nodes in the cell.
 - (ii) Each node checks the T-duplicate packets it carries. If there is more than one T-duplicate packet destined to the same destination, randomly pick one and drop the others.
- (4) **Receiving:** The unit square is divided into a regular lattice with n/M_2 cells. This step consists of $12D$ time slots. At each time slot, the nodes in good cells do the following at the minislot allocated to their cells.
 - (i) The nodes which contain deliverable packets randomly pick one deliverable packet and send a request to the corresponding destination.

- (ii) For each destination, it accepts only one request.
- (iii) The nodes whose requests are accepted transmit the deliverable packets to their destinations using the highway algorithm proposed in [17].

At the end of this step, all undelivered duplicate packets are dropped. Destinations use Raptor decoding to decode the received coded packets

Theorem 16: Consider Joint Coding–Scheduling Algorithm IV. Suppose D is both $\omega(1)$ and $o(\sqrt{n}/\log^2 n)$, and the delay constraint is $14D$. Then given any $\epsilon > 0$, there exists n_0 such that for any $n \geq n_0$, every data packet sent out can be recovered at the destination with probability $1 - \epsilon$, and furthermore

$$\lim_{T \rightarrow \infty} \Pr \left(\frac{\Lambda_i[T]}{T} \geq \left(\frac{W}{1400\sqrt{2}c_s C} \right)^4 \sqrt{\frac{D^2}{n}} \forall i \right) = 1.$$

Proof: Following the analysis of Theorem 14, it can be shown that the following events happen with high probability.

Node distribution: All cells are good during the entire super-time slot with high probability, i.e.,

$$\Pr(\mathcal{G}) \geq 1 - \frac{1}{n^2}. \quad (43)$$

Broadcasting: At least $3D/10$ coded packets from a source are successfully duplicated after the broadcasting with high probability, where a coded packet is said to be successfully duplicated if it has at least $M_1/3$ B-duplicate packets. Specifically, we have

$$\Pr \left(A_i \geq \frac{3}{10}D \mid \mathcal{G} \right) \geq 1 - \frac{1}{n^2}. \quad (44)$$

Transporting: At least $3D/40$ coded packets from a source are successfully transported after the transporting with high probability, where a coded packet is said to be successfully transported if it has at least $4M_1/5$ T-duplicate copies. Specifically, we have

$$\Pr \left(C_i \geq \frac{3}{40}D \mid \mathcal{G}, A_i \geq \frac{3}{10}D \right) \geq 1 - e^{-\frac{D}{3600}} - \frac{180}{\log n}. \quad (45)$$

Receiving: At least $D/40$ distinct coded packets from a source are delivered to its destination after the receiving. Specifically, we have

$$\Pr \left(B_i \geq \frac{D}{40} \mid \mathcal{G}, C_i \geq \frac{3}{40}D \right) \geq 1 - 2e^{-\frac{D}{1000}}. \quad (46)$$

Thus, the probability that the $D/50$ data packets are fully recovered in one super-time slot is at least

$$1 - \frac{2}{n^2} - e^{-\frac{D}{3600}} - \frac{180}{\log n} - 2e^{-\frac{D}{1000}},$$

and the theorem holds. \square

XI. ONE-DIMENSIONAL HYBRID RANDOM WALK MODELS, FAST AND SLOW MOBILES

In this section, we present the optimal delay–throughput tradeoffs of the one-dimensional hybrid random walk models.

The results can be established by following the analysis of the one-dimensional i.i.d. mobility models, where the technique difficulties caused by the random walk can be tackled by the Azuma–Hoeffding inequality as in the two-dimensional random walk models. The details are omitted here for brevity.

Theorem 17: Consider the one-dimensional hybrid random walk model and assume that Assumptions 1–3 hold. Then for $S = o(1)$ and $D = \omega(1/S^2)$, we have following results.

- (1) For fast mobiles

$$24WT \sqrt[3]{\frac{1}{\pi\Delta^2}} \sqrt[3]{n} (\sqrt[3]{D} + 1) \geq E[\Lambda[T]]. \quad (47)$$

When $S = o(1)$ and D is both $\omega(\max\{(\log^2 n) \log S / S^4, \sqrt[5]{n} \log n\})$ and $o(\sqrt{n} / \sqrt[3]{\log n})$, Joint Coding–Scheduling Algorithm III can be used to achieve a throughput the same as (47) except for a constant factor.

- (2) For slow mobiles

$$12WT \sqrt[4]{\frac{1}{\pi\Delta^2}} \sqrt[4]{n} (\sqrt{D} + 1) \geq E[\Lambda[T]]. \quad (48)$$

When $S = o(1)$ and D is both $\omega((\log^2 n) \log S / S^4)$ and $o(\sqrt{n} / \log^2 n)$, Joint Coding–Scheduling Algorithm IV can be used to achieve a throughput the same as (48) except for a constant factor.

XII. CONCLUSION

In this paper, we investigated the optimal delay–throughput tradeoff in mobile *ad hoc* networks. The optimal tradeoffs have been established under some conditions on delay D . When these conditions are not met, the optimal tradeoffs are still unknown in general. We would like to comment that the key to establishing the optimal delay–throughput tradeoff is to obtain $P_{i,j}(D, L)$, the probability that node i hits node j in one of D consecutive time slots given a hitting distance L . For example, under the two-dimensional hybrid random walk model, the upper bound was obtained under the condition $D = \omega(|\log S|/S^2)$ since it was the condition under which we established an upper bound on $P_{i,j}(D, L)$ (inequality (30)). Further, the maximum throughput was shown to be achievable under a more restrict condition $D = \omega((\log^2 n) \log S / S^6)$ since it was the condition under which we established a lower bound on $P_{i,j}(D, L)$ (inequality (65)). Thus, if we can find techniques to compute $P_{i,j}(D, L)$ without using the restricts on D , then the delay–throughput tradeoffs can be characterized more generally. This is a topic for future research.

APPENDIX A RAPTOR CODES

Raptor codes were proposed by Shokrollahi in [19] and they have low coding complexity. Suppose we have an erasure channel with erasure probability p . To transmit M source packets $\{S_1, \dots, S_M\}$ over this erasure channel, Raptor codes first generate coded packets as follows.

Raptor Encoding:

- (1) Use a right-regular low-definition parity-check (LDPC) code to encode the source packets and

generate $N = \lceil M/(1 - R) \rceil$ precoded packets, where $R = (1 + \epsilon/2)/(1 + \epsilon)$ for some $\epsilon > 0$.

- (2) Use Luby transform (LT) encoding [20] with modified robust solition distribution $\tilde{\Phi}$ [19] to generate coded packets.

After receiving $(1 + \epsilon)M$ coded packets, the source packets are decoded as follows.

Raptor Decoding:

- (1) Use LT decoding [20] to decode at least $(1 - \delta)N$ precoded packets, where $\delta = (\epsilon/4)/(1 + \epsilon)$.
- (2) Use brief propagation for the right-regular LDPC code to decode the M original packets.

We say M packets are successfully delivered if $\tilde{S}_i = S_i$ for $i = 1, \dots, M$. The following result is presented in [19].

Lemma 18: The receiver can correctly decode the M data packets with probability at least $1 - 1/(M^{a(\epsilon)})$ for some $a(\epsilon) > 0$ after it obtains $(1 + \epsilon)M$ coded packets generated by Raptor codes. The number of operations used for encoding and decoding is $O(M)$.

The algorithms proposed in this paper transmit $O(D)$ data packets in $\Theta(D)$ time slots. Thus, the time taken by Raptor encoding/decoding is $O(D)$, which is at most at the same order as the delay D . So we omit the delay due to coding in our analysis.

APPENDIX B

THROUGHPUT OF STATIC WIRELESS NETWORKS

The throughput of a random wireless network with n static nodes and n random S-D pairs is introduced by Gupta and Kumar [1]. They showed that the maximum throughput per S-D pair is $O(1/\sqrt{n})$, and proposed a scheduling scheme achieving a throughput of $\Theta(1/\sqrt{n \log n})$ per S-D pair. This $\log n$ gap was later closed by Franceschetti *et al.* in [17] where the authors showed a throughput of $\Theta(1/\sqrt{n})$ per S-D pair is achievable. The result is obtained under the physical interference models. However, it can be easily extended to the protocol model by using the same algorithm.

Lemma 19: In a random wireless network with n static nodes and n S-D pairs, a throughput of

$$\lambda = \frac{W}{c_s \sqrt{n}}$$

bits/time slot per S-D pair is achievable, where c_s is a positive constant independent of n . \square

Suppose the nodes use a common transmission radius $r = \Theta(1/n)$. The key idea of [17] is to construct $\Theta(n)$ disjoint paths traversing the network vertically and horizontally. These paths are called highways in [17], and a throughput of $\Theta(1/\sqrt{n})$ per S-D pair is achievable by transmitting data throughout these highways. We call this algorithm a highway algorithm.

APPENDIX C

PROBABILITY RESULTS

In this appendix, we present some standard results in probability for the reader’s convenience. In addition, we also present some variations of standard results which do not seem to be available in any book to best of our knowledge.

The following lemma is a standard result in probability, which we provide here for convenience.

Lemma 20: Let X_1, \dots, X_n be independent 0–1 random variables such that $E[\sum_i X_i] = \mu$. Then, the following Chernoff bounds hold:

$$\Pr \left(\sum_{i=1}^n X_i < (1 - \delta)\mu \right) \leq e^{-\delta^2 \mu / 2} \tag{49}$$

$$\Pr \left(\sum_{i=1}^n X_i > (1 + \delta)\mu \right) \leq e^{-\delta^2 \mu / 3}. \tag{50}$$

Proof: A detailed proof can be found in [21]. \square

The next lemmas are variations of standard balls-and-bins problems. However, we have not seen the results for the particular variation that we need in this paper. So we present the lemmas along with brief proofs below.

Lemma 21: Assume we have m bins. At each time, choose h bins and drop one ball in each of them. Repeat this n times. Using N_1 to denote the number of bins containing at least one ball, the following inequality holds for sufficiently large n :

$$\Pr(N_1 \leq (1 - \delta)m\tilde{p}_1) \leq 2e^{-\delta^2 m\tilde{p}_1/3} \tag{51}$$

where $\tilde{p}_1 = 1 - e^{-nh/m}$.

Proof: At each time, bin i receives a ball with probability h/m . We let κ_i denote the number of balls in bin i . Now consider a related balls-and-bins problem where the ball dropping procedure is replaced by a certain number of trials as dictated by a Poisson random variable. Specifically, define \tilde{n} to be a Poisson random variable with mean n , and repeat the ball dropping procedure \tilde{n} times. Let $\tilde{\kappa}_i$ denote the number of balls in bin i in this case. It is easy to see that $\{\tilde{\kappa}_i\}$ are i.i.d. Poisson random variables with mean nh/m . So we can conclude

$$\begin{aligned} & \Pr(N_1 \leq (1 - \delta)m\tilde{p}_1) \\ &= \Pr \left(\sum_{i=1}^m 1_{\kappa_i \geq 1} \leq (1 - \delta)m\tilde{p}_1 \right) \\ &\leq \Pr \left(\sum_{i=1}^m 1_{\tilde{\kappa}_i \geq 1} \leq (1 - \delta)m\tilde{p}_1 \mid \tilde{n} \geq n \right) \\ &\leq \frac{\Pr \left(\sum_{i=1}^m 1_{\tilde{\kappa}_i \geq 1} \leq (1 - \delta)m\tilde{p}_1 \right)}{\Pr(\tilde{n} \geq n)} \\ &= 2\Pr \left(\sum_{i=1}^m 1_{\tilde{\kappa}_i \geq 1} \leq (1 - \delta)m\tilde{p}_1 \right). \end{aligned}$$

Since

$$\Pr(1_{\tilde{\kappa}_i \geq 1} = 1) = \Pr(\tilde{\kappa}_i \geq 1) = 1 - e^{-nh/m} = \tilde{p}_1$$

from Lemma 20, we have

$$\Pr(N_1 \leq (1 - \delta)m\tilde{p}_1) \leq 2e^{-\delta^2 m\tilde{p}_1/3}. \tag{51}$$

The above idea of using a Poisson number of trials to bound the probability of the occurrence of an event in a fixed number of trials is called the Poisson heuristic in [21].

Lemma 22: Suppose n balls are independently dropped into m bins and one trash can. After a ball is dropped, the probability in the trash can is $1 - p$, and the probability in a specific bin is p/m . Using N_2 to denote the number of bins containing at least one ball, the following inequality holds for sufficiently large n :

$$\Pr(N_2 \leq (1 - \delta)m\tilde{p}_2) \leq 2e^{-\delta^2 m\tilde{p}_2/3} \quad (52)$$

where $\tilde{p}_2 = 1 - e^{-\frac{np}{m}}$.

Proof: Let κ_i denote the number of balls in bin i . Next define \tilde{n} to be a Poisson random variable with mean n . We consider the case such that \tilde{n} balls are independently dropped in m bins. Using $\tilde{\kappa}_i$ to be number of balls in bin i in this case, it is easy to see that $\{\tilde{\kappa}_i\}$ are i.i.d. Poisson random variables with mean $\frac{np}{m}$. So we have

$$\begin{aligned} & \Pr(N_2 \leq (1 - \delta)m\tilde{p}_2) \\ &= \Pr\left(\sum_{i=1}^m 1_{\kappa_i \geq 1} \leq (1 - \delta)m\tilde{p}_2\right) \\ &= \Pr\left(\sum_{i=1}^m 1_{\tilde{\kappa}_i \geq 1} \leq (1 - \delta)m\tilde{p}_2 \mid \tilde{n} \geq n\right) \\ &\leq \frac{\Pr\left(\sum_{i=1}^m 1_{\tilde{\kappa}_i \geq 1} \leq (1 - \delta)m\tilde{p}_2\right)}{\Pr(\tilde{n} \geq n)}. \end{aligned}$$

Since

$$\Pr(1_{\tilde{\kappa}_i \geq 1} = 1) = \Pr(\tilde{\kappa}_i \geq 1) = 1 - e^{-\frac{np}{m}} = \tilde{p}_2$$

from Lemma 20, we have

$$\Pr\left(\sum_{i=1}^m 1_{\tilde{\kappa}_i \geq 1} \leq (1 - \delta)m\tilde{p}_2\right) \leq e^{-\delta^2 m\tilde{p}_2/3}.$$

which implies for sufficiently large n

$$\begin{aligned} \Pr(N_2 \leq (1 - \delta)m\tilde{p}_2) &\leq \sqrt{3\pi n} e^{-\delta^2 m\tilde{p}_2/3} \\ &\leq 2e^{-\delta^2 m\tilde{p}_2/3}. \quad \square \end{aligned}$$

Next we introduce the Azuma–Hoeffding inequality.

Lemma 23: Suppose that X_1, \dots, X_n are independent random variables, and there exists a constant $c > 0$ such that $f(\mathbf{X}) = f(X_1, \dots, X_n)$ satisfies the following condition for any i and any set of values x_1, \dots, x_n and y_i :

$$\begin{aligned} & |f(x_1, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_n) \\ & \quad - f(x_1, \dots, x_{i-1}, y_i, x_{i+1}, \dots, x_n)| \leq c. \end{aligned}$$

Then we have

$$\Pr(|f(\mathbf{X}) - E[f(\mathbf{X})]| \geq \delta) \leq 2e^{-\frac{2\delta^2}{nc^2}}.$$

Proof: A detailed proof can be found in [21]. \square

APPENDIX D

PROOF OF INEQUALITIES (17), (18), AND (19)

Proof of Inequality (17): Let $\mathcal{B}_i[t]$ denote the event that node i broadcasts a coded packet at time slot t . So $\mathcal{B}_i[t]$ occurs when following two conditions hold:

- (1) the cell node i in is a good cell;
- (2) node i is selected to broadcast.

Since the nodes are uniformly randomly positioned, from the Chernoff bound we have

$$\Pr(\mathcal{B}_i[t]) \geq \frac{10}{11M} \left(1 - 2e^{-\frac{M}{300}}\right)$$

which implies that there exists \tilde{n}_1 such that for any $n \geq \tilde{n}_1$

$$\Pr(\mathcal{B}_i[t]) \geq \frac{8}{9} \frac{1}{M}.$$

Then from the Chernoff bound again, we have

$$\Pr\left(\sum_{t=1}^D 1_{\mathcal{B}_i[t]} \geq \frac{4}{5} \frac{D}{M}\right) \geq 1 - e^{-\frac{D}{300M}} \quad (53)$$

for $n \geq \tilde{n}_1$. Thus, with high probability, more than $4D/(5M)$ coded packets are broadcast, and each broadcast generates $9M/10$ copies.

Duplicate packets might be dropped at step (ii) of the broadcasting. We next calculate the number of duplicate packets of node i left after the broadcasting. Assume node i broadcasts \tilde{D}_i coded packets, so $\tilde{D}_i \leq D/M$. Then the number of duplicate packets left after the broadcasting is the same as the number of nonempty bins of the following balls-and-bins problem, where the bins represent the mobile nodes other than node i , and the balls represent the duplicate packets broadcast from node i .

Balls-and-Bins Problem: Assume we have $(n-1)$ bins. At each time slot, we select $9M/10$ bins and drop one ball in each of them. Repeat this \tilde{D}_i times.

Using N_1 to denote this number, from Lemma 21, we have

$$\Pr(N_1 \geq (1 - \delta)(n-1)\tilde{p}_1) \geq 1 - 2e^{-\delta^2(n-1)\tilde{p}_1/3}$$

where

$$\tilde{p}_1 = \left(1 - e^{-\frac{9\tilde{D}_i M}{10(n-1)}}\right).$$

Using the fact $1 - e^{-x} \geq x - x^2/2$ for any $x \geq 0$, we get

$$\begin{aligned} (n-1)\tilde{p}_1 &= (n-1) \left(1 - e^{-\frac{9\tilde{D}_i M}{10(n-1)}}\right) \\ &\geq \frac{9\tilde{D}_i M}{10} - \frac{81\tilde{D}_i^2 M^2}{100n - 100} \\ &\geq \frac{44}{49} \tilde{D}_i M \end{aligned}$$

where the last inequality holds for $n \geq \tilde{n}_2$ for some \tilde{n}_2 since $\tilde{D}_i M \leq D = o(n)$. Thus, choose $\delta = 1/50$ and we can conclude for $n \geq \tilde{n}_2$

$$\Pr\left(N_1 \geq \frac{22}{25} \tilde{D}_i M \mid \sum_{t=1}^D 1_{\mathcal{B}_i[t]} = \tilde{D}_i\right) \geq 1 - 2e^{-\frac{\tilde{D}_i M}{10000}}. \quad (54)$$

Recall a coded packet is said to be successfully duplicated if it has at least $4M/5$ copies at the end of the broadcasting. Inequality (54) implies for $n \geq \tilde{n}_2$

$$\Pr \left(A_i \geq \frac{4}{5} \tilde{D}_i \left| \sum_{t=1}^D 1_{\mathcal{B}_i[t]} = \tilde{D}_i \right. \right) \geq 1 - 2e^{-\frac{\tilde{D}_i M}{10000}}$$

since, otherwise, less than $22\tilde{D}_i M/25$ duplicate packets are left in the network. Thus, we can conclude that for $n \geq \tilde{n}_2$

$$\Pr \left(A_i \geq \frac{16}{25} \frac{D}{M} \left| \sum_{t=1}^D 1_{\mathcal{B}_i[t]} \geq \frac{4}{5} \frac{D}{M} \right. \right) \geq 1 - 2e^{-\frac{D}{20000}}. \quad (55)$$

Letting $n_1 = \max\{\tilde{n}_1, \tilde{n}_2\}$, inequality (17) follows from inequalities (53) and (55) for $n \geq n_1$.

Proof of Inequality (18): Assume coded packets $\{(i, 1), \dots, (i, 16D/(25M))\}$ are successfully duplicated. We let $\mathcal{D}_{(i,k)}[t]$ denote the event that coded packet (i, k) is delivered at time slot t . Then $\mathcal{D}_{(i,k)}[t]$ will definitely occur if both the following conditions hold.

- (1) One and only one duplicate packet of (i, k) becomes a deliverable packet. Let $\mathcal{D}_{(i,k)}^1[t]$ denote this event. Assume the duplicate packet is (i, k, j) , i.e., node j contains packet (i, k) .
- (2) There are no other deliverable packets in the cell containing node j except packet (i, k, j) and one possible duplicate packet to node j carried by node $i + 1$. Let $\mathcal{D}_{(i,k)}^2[t]$ denote this event.

Note that duplicate packets of node i are carried by different nodes, and their mobilities are independent. Now assume there are $\tilde{M}_{(i,k)}$ copies of (i, k) in the entire network, then

$$\Pr \left(\mathcal{D}_{(i,k)}^1[t] \right) = \frac{\tilde{M}_{(i,k)} M}{n} \left(1 - \frac{M}{n} \right)^{\tilde{M}_{(i,k)} - 1}.$$

Note that $(1 - M/n)^{\tilde{M}_{(i,k)} - 1} \geq 1 - (\tilde{M}_{(i,k)} - 1)M/n$ and $\tilde{M}_{(i,k)} \geq 4M/5$ if (i, k) is successfully duplicated. So for a successfully duplicated packet, there exists \tilde{n}_3 such that for any $n \geq \tilde{n}_3$

$$\Pr \left(\mathcal{D}_{(i,k)}^1[t] \right) \geq \frac{7M^2}{10n}.$$

Suppose we have \bar{M} nodes in the cell containing node j ; from the Chernoff bound, we have

$$\Pr \left(\bar{M} \leq \frac{11}{10} M \right) \geq 1 - e^{-\frac{M}{300}}.$$

Note that condition (2) is equivalent to the following event: Given node j and node $i + 1$ in the cell, no more deliverable packets appear when we put another $\bar{M} - 2$ nodes into the cell. Now given K nodes in the cell, the probability that no more deliverable packet appears when we put another node is at least

$$\left(1 - \frac{2KD}{n - K} \right).$$

This holds due to the following two facts.

- (i) The new node should not be the destination of any duplicate packets already in the cell (there are at most KD duplicate packets already in the cell).

- (ii) The duplicate packets carried by the new node are not destined for any of the existing K nodes. Note that each source has no more than D duplicate packets, so there are at most KD nodes which carry the duplicate packet toward the K existing nodes.

Note that $\lim_{n \rightarrow \infty} M = \infty$, so there exists \tilde{n}_4 such that for any $n \geq \tilde{n}_4$

$$\begin{aligned} & \Pr \left(\mathcal{D}_{(i,k)}^2[t] \left| \mathcal{D}_{(i,k)}^1[t] \right. \right) \\ & \geq \left(1 - e^{-\frac{M}{300}} \right) \prod_{K=2}^{\frac{11M}{10} - 1} \left(1 - \frac{2KD}{n - K} \right) \\ & \geq \left(1 - e^{-\frac{M}{300}} \right) \left(1 - \frac{22MD}{10n - 11M} \right)^{\frac{11M}{20}} \\ & \geq \frac{3}{11}. \end{aligned}$$

So we can conclude that for any $n \geq \max\{\tilde{n}_3, \tilde{n}_4\}$

$$\Pr \left(\mathcal{D}_{(i,k)}[t] \right) \geq \frac{21M^2}{110n} = \frac{21}{110D} \quad (56)$$

which implies that at each time slot, a successfully duplicated packet (i, k) will be delivered with probability at least $21/(110D)$. Note at each time slot, only one coded packet can be delivered to the destination of node i . So the number of distinct coded packets delivered to the destination of node i is the same as the number of nonempty bins of following balls-and-bins problem, where the bins represent the distinct coded packets, the balls represent successful deliveries, and a ball is dropped in a specific bin means the corresponding coded packet is delivered to the destination.

Balls-and-Bins Problem: Suppose we have $16D/(25M)$ bins and one trash can. At each time slot, we drop a ball. Each bin receives the ball with probability $21/(110D)$, and the trash can receives the ball with probability $1 - p$, where

$$p = \frac{21}{110D} \times \frac{16D}{25M} = \frac{168}{1375} \frac{1}{M}.$$

Repeat this $5D$ times, i.e., $5D$ balls are dropped.

Let N_2 denote nonempty bins of the above balls-and-bins problem and choose $\delta = 1/6$. From Lemma 21, we have

$$\Pr \left(N_2 \geq \frac{7}{25} \frac{D}{M} \right) \geq 1 - 2e^{-\frac{D}{180M}}$$

and inequality (18) holds for $n \geq n_2$, where $n_2 = \max\{\tilde{n}_3, \tilde{n}_4\}$.

Proof of Inequality (19): Inequality (19) follows from Lemma 18 on the error probability of Raptor codes.

APPENDIX E PROOF OF INEQUALITY (28)

Proof: Assume that coded packets $(i, 1), \dots, (i, 4D/5)$ are successfully duplicated. Note that duplicate packets from a common source are carried by different nodes; and the mobilities of those nodes are independent. So (i, k) will be definitely delivered to its destination if both the following conditions hold.

- (1) A copy of (i, k) is the only deliverable packet for destination $i+1$ in time slot t . Let $\mathcal{D}_{(i,k)}^1[t]$ denote this event; and assume that the duplicate packet is in node j .
- (2) Node j has no other deliverable packet, and the cell containing node j is good. Let $\mathcal{D}_{(i,k)}^2[t]$ denote this event.

Let $\tilde{M}_{(i,k)}$ denote the number of copies of (i, k) . Since each source has at most $9M_1D/10$ duplicate packets in the network, we have

$$\Pr\left(\mathcal{D}_{(i,k)}^1[t]\right) \geq \frac{\tilde{M}_{(i,k)}M_2}{n} \left(1 - \frac{M_2}{n}\right)^{\frac{9}{10}M_1D-1}.$$

It is easy to verify that

$$\lim_{n \rightarrow \infty} \left(1 - \frac{M_2}{n}\right)^{\frac{9}{10}M_1D-1} = e^{-0.9}.$$

Thus, for successfully duplicated packet (i, k) , i.e., $\tilde{M}_{(i,k)} \geq 4M_1/5$, we can conclude that

$$\Pr\left(\mathcal{D}_{(i,k)}^1[t]\right) \geq \frac{4M_1M_2}{5en} \quad (57)$$

holds for sufficiently large n . Note that each node carries at most DM_1 duplicate packets, so we further have

$$\Pr\left(\mathcal{D}_{(i,k)}^2[t] \mid \mathcal{D}_{(i,k)}^1[t]\right) \geq \left(1 - \frac{M_2}{n}\right)^{DM_1-1} - 2e^{-\frac{M_2}{300}} \quad (58)$$

where $(1 - M_2/n)^{DM_1-1}$ is the lower bound on the probability that all packets in node j except (i, k, j) are undeliverable, and $1 - 2e^{-\frac{M_2}{300}}$ is the probability that the cell is good.

From inequalities (57) and (58), we can conclude that for sufficiently large n

$$\begin{aligned} & \Pr\left(\mathcal{D}_{(i,k)}^1[t], \mathcal{D}_{(i,k)}^2[t]\right) \\ &= \Pr\left(\mathcal{D}_{(i,k)}^1[t]\right) \Pr\left(\mathcal{D}_{(i,k)}^2[t] \mid \mathcal{D}_{(i,k)}^1[t]\right) \\ &\geq \frac{M_1M_2}{12n} = \frac{1}{12D}. \end{aligned}$$

Inequality (28) can be proved by the balls-and-bins argument used to show inequality (18). \square

APPENDIX F PROPERTIES OF RANDOM WALK

Consider following two random walks.

- (1) **One-dimensional random walk:** A random walk on a circle with unit length and $1/\tilde{S}$ points. At each time slot, a node moves to one point left, one point right or does not move with equal probability as in Fig. 12.
- (2) **Two-dimensional random walk:** A random walk on a unit torus with $1/\tilde{S}^2$ points. At each time slot, a node moves to one of eight neighbors or does not move with equal probability as in Fig. 13.

We introduce following definitions.

- Transition matrix \mathbf{P} : $\mathbf{P} = [P_{\mathbf{i},\mathbf{j}}]$ where $P_{\mathbf{i},\mathbf{j}}$ is the probability of moving from point \mathbf{i} to point \mathbf{j} .
- Stationary distribution Π : A vector which satisfies the equation $\Pi\mathbf{P} = \Pi$.

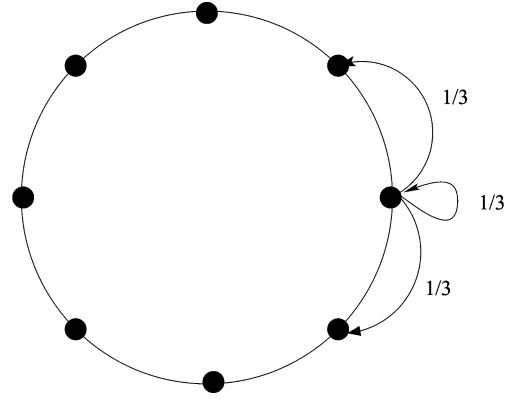


Fig. 12. One-dimensional random walk.

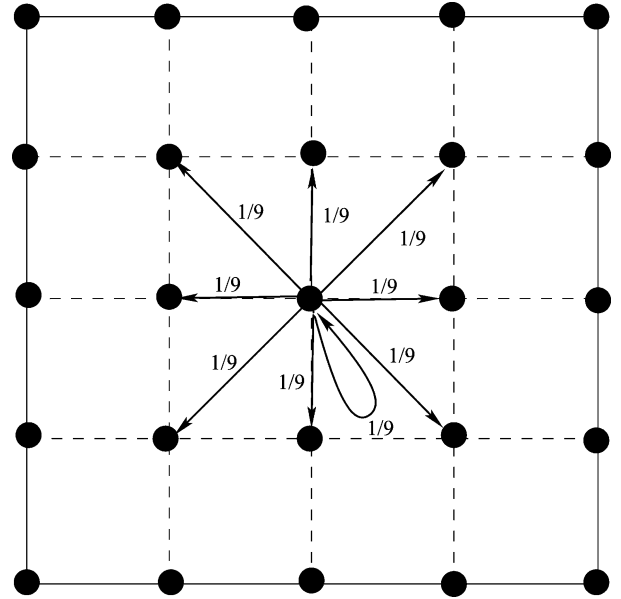


Fig. 13. Two-dimensional random walk.

- Hitting time $T_h(\mathbf{i}, \mathbf{j})$: Time taken for a node to move from point \mathbf{i} to point \mathbf{j} .
- Mixing time T_m

$$T_m = \inf_t \sup_{\mathbf{i}} \sum_{\mathbf{j}} |P_{\mathbf{i},\mathbf{j}}^t - \Pi_{\mathbf{j}}| \leq \tilde{S}^4$$

where $P_{\mathbf{i},\mathbf{j}}^t$ is the (\mathbf{i}, \mathbf{j}) th entry of \mathbf{P}^t .

Lemma 24: For the one-dimensional random walk, we have

- $E[T_h(\mathbf{i}, \mathbf{j})] = O(1/\tilde{S}^2)$;
- $T_m = O(|\log \tilde{S}|/\tilde{S}^2)$.

For the two-dimensional random walk, we have

- $E[T_h(\mathbf{i}, \mathbf{j})] = O(|\log \tilde{S}|/\tilde{S}^2)$;
- $T_m = O(|\log \tilde{S}|/\tilde{S}^2)$.

Proof: Please refer to [22] for the one-dimensional random walk. The proof of the hitting time of the two-dimensional random walk is presented in Lemma 13 in [7], and the mixing time result holds since the two-dimensional random walk can be regarded as two independent one-dimensional random walks. \square

Lemma 25: Let $N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D]$ denote the number of visits to point \mathbf{j} in D time slots starting from point \mathbf{i} and ending at point \mathbf{k} . If $D = \omega(|\log \hat{S}|/\hat{S})$, we have

$$\frac{9}{10}D\hat{S} \leq E[N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D]] \leq \frac{11}{10}D\hat{S} \quad (59)$$

where $\hat{S} = \tilde{S}$ for the one-dimensional random walk and $\hat{S} = \tilde{S}^2$ for the two-dimensional random walk. Furthermore, if $D = \kappa\alpha|\log \hat{S}|/\hat{S}^3$ where $\kappa = \omega(1)$, then we have

$$\Pr\left(\frac{6}{5}D\hat{S} \geq N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D] \geq \frac{4}{5}D\hat{S}\right) \geq 1 - 2e^{-\frac{\alpha}{625}}. \quad (60)$$

Proof: First we have

$$T_h(\mathbf{i}, \mathbf{j}) + \sum_{l=1}^{N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D]-1} T_h^l(\mathbf{j}, \mathbf{j}) + T_h(\mathbf{j}, \mathbf{k}) = D$$

where $T_h^l(\mathbf{j}, \mathbf{j})$ is the time duration between l th visits to point \mathbf{j} and $(l+1)$ th visits to point \mathbf{j} . Taking the expectation on both sides, we have

$$E[T_h(\mathbf{i}, \mathbf{j})] + E[N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D]]E[T_h(\mathbf{j}, \mathbf{j})] - E[T_h(\mathbf{j}, \mathbf{j})] + E[T_h(\mathbf{j}, \mathbf{k})] = D$$

which implies

$$E[N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D]] = \frac{D - E[T_h(\mathbf{i}, \mathbf{j})] - E[T_h(\mathbf{j}, \mathbf{k})] + E[T_h(\mathbf{j}, \mathbf{j})]}{E[T_h(\mathbf{j}, \mathbf{j})]}.$$

Inequality (59) follows from the facts that

$$E[T_h(\mathbf{i}, \mathbf{j})] = O(|\log \hat{S}|/\hat{S}) \text{ and } E[T_h(\mathbf{j}, \mathbf{j})] = 1/\hat{S}.$$

Next, let $\mathbf{x}[t]$ denote the position of the node at time slot t , $\tilde{\mathbf{X}}$ denote $\{\mathbf{x}[t]\}$ for $t = 1, \frac{D\hat{S}^2}{\alpha} + 1, \frac{2D\hat{S}^2}{\alpha} + 1, \dots, D - \frac{D\hat{S}^2}{\alpha}, D$, and \mathbf{X}_m denote $\{\mathbf{x}[t]\}$ for $t = \frac{mD\hat{S}^2}{\alpha} + 1, \frac{mD\hat{S}^2}{\alpha}, \dots, \min\left\{D, \frac{(m+1)D\hat{S}^2}{\alpha}\right\}$ where $m=0, \dots, \alpha/\hat{S}^2-1$. Further, let $N_{\mathbf{j},\tilde{\mathbf{X}}}[D]$ denote the number of visits to point \mathbf{j} given $\tilde{\mathbf{X}}$. It is easy to see that for any $\tilde{\mathbf{X}}$ there exists a function $f_{\tilde{\mathbf{X}}}$ such that

$$N_{\mathbf{j},\tilde{\mathbf{X}}}[D] = f_{\tilde{\mathbf{X}}}(\mathbf{X}_1, \dots, \mathbf{X}_{\alpha/\hat{S}^2-1})$$

where $\{\mathbf{X}_m\}$ are mutually independent given $\tilde{\mathbf{X}}$. Note that \mathbf{X}_m contains the position information from time slot $mD\hat{S}^2/\alpha + 1$ to time slot $(m+1)D\hat{S}^2/\alpha$, so

$$\left|f_{\tilde{\mathbf{X}}}(\mathbf{X}_0, \dots, \mathbf{X}_{m-1}, \mathbf{X}_m, \mathbf{X}_{m+1}, \dots, \mathbf{X}_{\alpha/\hat{S}^2-1}) - f_{\tilde{\mathbf{X}}}(\mathbf{X}_0, \dots, \mathbf{X}_{m-1}, \mathbf{Y}_m, \mathbf{X}_{m+1}, \dots, \mathbf{X}_{\alpha/\hat{S}^2-1})\right| \leq \frac{D\hat{S}^2}{\alpha}.$$

Next note that $D\hat{S}^2/\alpha = \omega(|\log \hat{S}|/\hat{S})$, so from inequality (59), we can conclude that for any \mathbf{i}, \mathbf{j} , and \mathbf{k}

$$\frac{9}{10} \frac{D\hat{S}^3}{\alpha} \leq E\left[N_{\mathbf{i},\mathbf{j},\mathbf{k}}\left[\frac{D\hat{S}^2}{\alpha}\right]\right] \leq \frac{11}{10} \frac{D\hat{S}^3}{\alpha}$$

which implies that

$$\frac{9}{10}D\hat{S} \leq E\left[N_{\mathbf{j},\tilde{\mathbf{X}}}[D]\right] \leq \frac{11}{10}D\hat{S}$$

holds for any $\tilde{\mathbf{X}}$. Then from the Azuma-Hoeffding inequality (Lemma 23), we have that

$$\Pr\left(\left|N_{\mathbf{j},\tilde{\mathbf{X}}}[D] - E[N_{\mathbf{j},\tilde{\mathbf{X}}}[D]]\right| \leq \frac{9}{100}D\hat{S}\right) \geq 1 - 2e^{-\frac{\alpha}{625}}$$

for any $\tilde{\mathbf{X}}$, and inequality (60) holds. \square

APPENDIX G

PROOF OF INEQUALITY (30)

Let N_b^{rw} denote the number of time slots, from t_b+1 to t_b+D , at which node c_b and d_b are in the same RW-cell or neighboring RW-cells. Then for any $L \in [0, S/\sqrt{\pi}]$, we have

$$\begin{aligned} \Pr(\tilde{L}_b \leq L) &= \sum_{K=1}^D \Pr(\tilde{L}_b \leq L | N_b^{\text{rw}} = K) \Pr(N_b^{\text{rw}} = K) \\ &\leq \sum_{K=1}^D \left(1 - \left(1 - \frac{\pi L^2}{S^2}\right)^K\right) \Pr(N_b^{\text{rw}} = K) \\ &= 1 - E\left[\left(1 - \frac{\pi L^2}{S^2}\right)^{N_b^{\text{rw}}}\right] \\ &\leq 1 - \left(1 - \frac{\pi L^2}{S^2}\right)^{E[N_b^{\text{rw}}]} \end{aligned}$$

where the first inequality follows from the fact that the node position within an RW-cell is randomly uniformly selected, and the last inequality follows from the Jensen's inequality.

Next we consider $E[N_b^{\text{rw}}]$. Let $(U_i^x(t), U_i^y(t))$ denote the RW-cell in which node i is at time slot t , and $(V_i^x(t), V_i^y(t))$ denote the displacement of node i at time slot t , i.e.,

$$V_i^x(t) = \begin{cases} 1, & \text{w.p. } \frac{1}{3} \\ 0, & \text{w.p. } \frac{1}{3} \\ -1, & \text{w.p. } \frac{1}{3} \end{cases} \text{ and } V_i^y(t) = \begin{cases} 1, & \text{w.p. } \frac{1}{3} \\ 0, & \text{w.p. } \frac{1}{3} \\ -1, & \text{w.p. } \frac{1}{3} \end{cases}.$$

It is easy to see that

$$\begin{aligned} U_i^x(t) &= \left[\left(U_i^x(0) + \sum_{m=1}^{t-1} V_i^x(m)\right) \bmod \frac{1}{S}\right] + 1 \\ U_i^y(t) &= \left[\left(U_i^y(0) + \sum_{m=1}^{t-1} V_i^y(m)\right) \bmod \frac{1}{S}\right] + 1. \end{aligned}$$

Further, let $(U_{i-j}^x(t), U_{i-j}^y(t))$ denote the relative position of node i from node j , i.e.,

$$\begin{aligned} U_{i-j}^x(t) &= \left(U_{i-j}^x(0) + \sum_{m=1}^{t-1} \tilde{V}_{i-j}^x(m)\right) \bmod \frac{1}{S}; \\ U_{i-j}^y(t) &= \left(U_{i-j}^y(0) + \sum_{m=1}^{t-1} \tilde{V}_{i-j}^y(m)\right) \bmod \frac{1}{S}, \end{aligned}$$

where

$$\tilde{V}_{i-j}^x(t) = V_i^x(t) - V_j^x(t) = \begin{cases} 2, & \text{w.p. } \frac{1}{c_i - c_j} \\ 1, & \text{w.p. } \frac{1}{c_i - c_j} \\ 0, & \text{w.p. } \frac{1}{c_i - c_j} \\ -1, & \text{w.p. } \frac{1}{c_i - c_j} \\ -2, & \text{w.p. } \frac{1}{c_i - c_j} \end{cases}$$

and

$$\tilde{V}_{i-j}^y(t) = V_i^y(t) - V_j^y(t) = \begin{cases} 2, & \text{w.p. } \frac{1}{c_i - c_j} \\ 1, & \text{w.p. } \frac{1}{c_i - c_j} \\ 0, & \text{w.p. } \frac{1}{c_i - c_j} \\ -1, & \text{w.p. } \frac{1}{c_i - c_j} \\ -2, & \text{w.p. } \frac{1}{c_i - c_j} \end{cases}$$

So $(U_{i-j}^x(t), U_{i-j}^y(t))$ is the consequence of random walk $(\tilde{V}_{i-j}^x(m), \tilde{V}_{i-j}^y(m))$ with initial position

$$(U_{i-j}^x(0), U_{i-j}^y(0)) = (U_i^x(0) - U_j^x(0), U_i^y(0) - U_j^y(0)).$$

Note that node c_b and node d_b are in the same RW-cell if $(U_{c_b-d_b}^x(t), U_{c_b-d_b}^y(t)) = (0, 0)$, and in neighboring RW-cells if

$$(U_{c_b-d_b}^x(t), U_{c_b-d_b}^y(t)) \in \{(0, 1), (1, 0), (1, 1), (0, 1/S-1), (1/S-1, 0), (1/S-1, 1/S-1)\}.$$

Similar to the argument in Lemma 25, we can conclude that for $D = \omega(|\log S|/S^2)$

$$E[N_b^{\text{rw}}] \leq \frac{99}{10} S^2 D$$

which implies that

$$\Pr(\tilde{L}_b \leq L) \leq 1 - \left(1 - \frac{\pi L^2}{S^2}\right)^{\frac{99}{10} S^2 D} \leq 36L^2 D.$$

APPENDIX H

PROOF OF INEQUALITIES (31), (32), AND (33)

Proof of Inequality (31): Since $D = o(n/\log^2 n)$ implies $M = \omega(\log n)$. Inequality (31) can be obtained from the Chernoff bound and union bound.

Proof of Inequality (32): Consider the broadcasting. Note that when \mathcal{G} occurs, node i is selected to broadcast with probability at least $10/(11M)$ at each time slot. Let $\mathcal{B}_i[t]$ denote the event that node i is selected to broadcast in time slot t . From the Chernoff bound, we have

$$\Pr\left(\sum_{t=1}^D 1_{\mathcal{B}_i[t]} \geq \frac{9}{11} \frac{D}{M} \mid \mathcal{G}\right) \geq 1 - e^{-\frac{D}{600M}}. \quad (61)$$

So node i broadcasts $9D/(11M)$ coded packets with a high probability. Each coded packet is broadcast to $9M/10$ relay nodes.

According to Step (2) (ii) of Joint Coding–Scheduling Algorithm I, each relay node keeps at most one packet for each source. Consider duplicate packet (i, k, j) . It could be dropped

if node j is in the same cell as node i and node i is selected to broadcast. Thus, the probability that (i, k, j) is dropped is at most

$$\frac{11}{10} \frac{DM}{n} \times \frac{10}{9} \frac{1}{M} = \frac{11}{9} \frac{D}{n} \quad (62)$$

due to the following two facts.

- (1) Let $\mathcal{H}_{ji}[D]$ denote the event that node j is in the same cell as node i in at least one of D consecutive time slots. Similar to (30), it can be shown that

$$\Pr(\mathcal{H}_{ji}[D]) \leq \frac{11}{10} \frac{DM}{n} \quad (63)$$

under the delay constraint given in the theorem.

- (2) When \mathcal{G} occurs, node i is selected to broadcast with probability at most $10/(9M)$ at each time slot.

Now suppose source i broadcasts \tilde{D}_i coded packets, so $9M\tilde{D}_i/10$ duplicate copies are generated. Let \tilde{N}_i^d denote the number of duplicate packets of node i dropped in the broadcasting. From the Markov inequality and inequality (62), we have

$$\begin{aligned} & \Pr\left(\tilde{N}_i^d \geq \frac{M\tilde{D}_i}{50} \mid \mathcal{G}, \sum_{t=1}^D 1_{\mathcal{B}_i[t]} = \tilde{D}_i\right) \\ & \leq \frac{E\left[\tilde{N}_i^d \geq \frac{M\tilde{D}_i}{50} \mid \mathcal{G}, \sum_{t=1}^D 1_{\mathcal{B}_i[t]} = \tilde{D}_i\right]}{\frac{M\tilde{D}_i}{50}} \\ & \leq \frac{\frac{9M\tilde{D}_i}{10} \times \frac{11D}{9n}}{\frac{M\tilde{D}_i}{50}} \\ & = \frac{55D}{n} \end{aligned}$$

which implies

$$\Pr\left(A_i \geq \frac{4}{5} \tilde{D}_i \mid \mathcal{G}, \sum_{t=1}^D 1_{\mathcal{B}_i[t]} = \tilde{D}_i\right) \geq 1 - \frac{55D}{n} \quad (64)$$

since otherwise, more than $M\tilde{D}_i/50$ duplicate packets would be dropped. Inequality (32) follows from inequalities (64) and (61).

Proof of Inequality (33): We group every $3D/\log D$ time slots into big time slots, named as b-time slot and indexed by t_b , and then divide every b-time slot into three equal parts, indexed by $t_{b,1}$, $t_{b,2}$, and $t_{b,3}$ as in Fig. 14. We first calculate the probability that coded packet (i, k) is delivered in $t_{b,2}$. Let $\mathcal{H}_{(i,k)}[t_{b,2}]$ denote the event that at least one copy of packet (i, k) becomes deliverable in $t_{b,2}$. If (i, k) is in at least $4M/5$ relay nodes, we can obtain

$$\begin{aligned} \Pr(\mathcal{H}_{(i,k)}[t_{b,2}]) & \geq 1 - \left(\left(1 - \frac{M}{nS^2}\right)^{\frac{4D S^2}{5 \log D}} + \frac{1}{n} \right)^{\frac{4M}{5}} \\ & \geq \frac{3}{5} \frac{1}{\log D} \end{aligned} \quad (65)$$

due to the following facts.

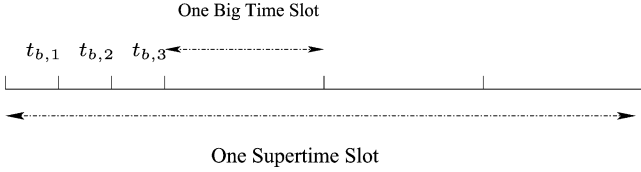


Fig. 14. Division of a super-time slot.

- (1) Given $D/\log D = \omega(\log n |\log S|/S^6)$, from Lemma 25 provided in Appendix F we know that with probability at least $1 - 1/n$, two nodes are in the same RW-cell for at least $4DS^2/(5 \log D)$ time slots.
- (2) Given two nodes are in the same RW-cell, the probability that they are in the same cell is $M/(nS^2)$.

Next note that the duration of $t_{b,1}$ and $t_{b,3}$ are of a larger order than the mixing time of the random walk. From the definition of the mixing time, we have that at any time slot belonging to $t_{b,2}$, the nodes are almost uniformly distributed in the unit square. Let $\mathcal{D}_{(i,k)}[t_{b,2}]$ denote the event that coded packet (i, k) is delivered to its destination in $t_{b,2}$. Following the argument used to show inequality (56), we have

$$\Pr(\mathcal{D}_{(i,k)}[t_{b,2}]) \geq \frac{3}{20 \log D}. \quad (66)$$

Now let \mathbf{x}_t denote the positions of the nodes at time slot t , and

$$\tilde{\mathbf{X}} = \{\mathbf{x}_t\}_{t=\frac{3(k-1)D}{\log D}+1}$$

for $k = 1, \dots, 5 \log D/3$. Also, let $\mathcal{D}_{(i,k)}$ denote the event that (i, k) is delivered in the receiving. It is easy to see that $\mathcal{D}_{(i,k)}$ occurs if $\mathcal{D}_{(i,k)}[t_{b,2}]$ occurs for some $t_{b,2} \in \{1, \dots, 5 \log D/3\}$. Note that $\{\mathcal{D}_{(i,k)}[t_{b,2}]\}_{t_{b,2}}$ are mutually independent given $\tilde{\mathbf{X}}$, so from inequality (66), we have

$$\Pr(\mathcal{D}_{(i,k)} | \tilde{\mathbf{X}}) \geq 1 - \left(1 - \frac{3}{20 \log D}\right)^{5 \log D/3} \geq \frac{1}{5}.$$

Further, since $B_i \geq \sum_{k=1}^{A_i} 1_{\mathcal{D}_{(i,k)}}$, we can conclude that

$$E \left[B_i \mid \tilde{\mathbf{X}}, A_i \geq \frac{16 D}{25 M} \right] \geq \frac{16 D}{125 M}. \quad (67)$$

We next bound the number of distinct coded packets deliverable in t_b . Similar to inequality (65), we have

$$\Pr(\mathcal{H}_{(i,k)}[t_b]) \leq \frac{3}{\log D}.$$

Note that no two duplicate packets from node i are in one relay node, so $\{\mathcal{H}_{(i,k)}[t_b]\}_k$ are mutually independent. From the Chernoff bound, we have

$$\Pr \left(\sum_{k=1}^{D/M} 1_{\mathcal{H}_{(i,k)}[t_b]} \leq \frac{16 D}{5 M \log D} \right) \geq 1 - e^{-\frac{D}{400 M \log D}}.$$

Let $\tilde{\mathcal{F}}_i$ denote the event that node i obtains no more than $16D/(5M \log D)$ coded packets at each b-time slot in the receiving. From the union bound, we have that for sufficiently large n

$$\begin{aligned} \Pr(\tilde{\mathcal{F}}_i) &\geq 1 - \left(\frac{5}{3} \log D\right) \left(e^{-\frac{D}{400 M \log D}}\right) \\ &\geq 1 - e^{-\frac{D}{500 M \log D}}. \end{aligned} \quad (68)$$

Now let $B_i(\tilde{\mathbf{X}}, A_i, \mathcal{F}_i)$ denote the number of distinct coded packets delivered to the destination of node i given $(\tilde{\mathbf{X}}, A_i, \mathcal{F}_i)$, and \mathbf{X}_{t_b} denote an $n \times (3D/\log D)$ matrix where the (i, t) entry is the position of node i at the t th time slot of b-time slot t_b . It is easy to see that the value of $B_i(\tilde{\mathbf{X}}, A_i, \mathcal{F}_i)$ is determined by $\{\mathbf{X}_{t_b}\}$, i.e., there exists a function $f_{(\tilde{\mathbf{X}}, A_i, \mathcal{F}_i)}$ such that

$$B_i(\tilde{\mathbf{X}}, A_i, \mathcal{F}_i) = f_{(\tilde{\mathbf{X}}, A_i, \mathcal{F}_i)}(\mathbf{X}_1, \dots, \mathbf{X}_{5 \log D/3}).$$

From the definition of \mathcal{F}_i , function $f_{(\tilde{\mathbf{X}}, A_i, \mathcal{F}_i)}$ satisfies the following condition:

$$\begin{aligned} &\left| f_{(\tilde{\mathbf{X}}, A_i, \mathcal{F}_i)}(\mathbf{X}_1, \dots, \mathbf{X}_{t_b-1}, \mathbf{X}_{t_b}, \mathbf{X}_{t_b+1}, \dots, \mathbf{X}_{5 \log D/3}) \right. \\ &\quad \left. - f_{(\tilde{\mathbf{X}}, A_i, \mathcal{F}_i)}(\mathbf{X}_1, \dots, \mathbf{X}_{t_b-1}, \mathbf{Y}_{t_b}, \mathbf{X}_{t_b+1}, \dots, \mathbf{X}_{5 \log D/3}) \right| \\ &\leq \frac{16 D}{5 M \log D}. \end{aligned} \quad (69)$$

It is easy to see that $\{\mathbf{X}_{t_b}\}$ are mutually independent given $(\tilde{\mathbf{X}}, A_i, \tilde{\mathcal{F}}_i)$. Then invoking Azuma–Hoeffding inequality provided in Appendix A, we can conclude that

$$\begin{aligned} \Pr \left(B_i(\tilde{\mathbf{X}}, A_i, \mathcal{F}_i) \geq E \left[B_i \mid \tilde{\mathbf{X}}, A_i, \tilde{\mathcal{F}}_i \right] - \frac{1}{125} \frac{D}{M} \right) \\ \geq 1 - 2e^{-\frac{\log D}{8500}} \end{aligned} \quad (70)$$

holds for any $\tilde{\mathbf{X}}$ and A_i . Inequality (33) follows from inequalities (67), (68), and (70).

APPENDIX I

PROOF OF INEQUALITIES (40) AND (41)

Proof of Inequality (40): Assume that \mathcal{G} occurs, then at each time slot, node i is selected with probability $10/(11M)$. Note that there are $\sqrt{n/M}$ cells on the orbit of node i , and node i is uniformly randomly positioned in one of the cells. Thus, the number of coded packets broadcast by node i is equal to the number of nonempty bins of following balls-and-bins problem.

Balls-and-Bins Problem: Suppose we have $\sqrt{n/M}$ bins and one trash can. At each time slot, we drop a ball. Each bin receives the ball with probability $10/(11\sqrt{nM})$, and the trash can receives the ball with probability $1 - 10/(11M)$. Repeat this D times, i.e., D balls are dropped.

From Lemma 22, we have

$$\Pr \left(\sum_{k=1}^D 1_{B_i[t]} \geq \frac{9D}{11M} \mid \mathcal{G} \right) \geq 1 - e^{-\frac{3D}{1000M}}.$$

We say two nodes are competitive with each other if the orbits of their destinations are contained in the same rectangle, so each node has $\sqrt{Mn}-1$ competitive nodes. Suppose that node i is an H-node and node j is a V-node. Let $\tilde{N}_{i,j}^c(t)$ denote the number of node i 's competitive nodes in the V-rectangle containing the orbit of node j at time slot t . Since nodes are uniformly, randomly positioned on their orbits, from the Chernoff bound, we have

$$\Pr \left(\tilde{N}_{i,j}^c(t) \leq \frac{11}{10} M \right) \geq 1 - e^{-\frac{M}{300}}. \quad (71)$$

Now consider B-duplicate packet (i, k, j) and assume that node z , a competitive of node i , is in the V-rectangle containing the orbit of node i . Then (i, k, j) might be dropped when it is in the same cell as node z , and node z is selected to broadcast. The probability of this event is at most

$$\sqrt{\frac{M}{n}} \times \frac{10}{9M}. \quad (72)$$

From (71), (72), and the union bound, we can conclude that the probability that (i, k, j) is dropped at time slot t is at most

$$e^{-\frac{M}{300}} + \frac{11}{10}M \times \sqrt{\frac{M}{n}} \times \frac{10}{9M} = e^{-\frac{M}{300}} + \frac{11}{9} \sqrt{\frac{M}{n}} \quad (73)$$

which implies that the probability of (i, k, j) dropped in the broadcasting is at most

$$1 - \left(1 - e^{-\frac{M}{300}} - \frac{11}{9} \sqrt{\frac{M}{n}}\right)^D \leq De^{-\frac{M}{300}} + \frac{11}{9M}.$$

Inequality (40) follows from above inequality and the Markov inequality.

Proof of Inequality (41): Consider an H-node i . Let $C_{(i,k)}$ denote the number of B-duplicate packets which are contained in the V-rectangle where (i, k) broadcast, and are destined to the same H-rectangle as node i . Note the following facts.

- (1) Each node has $\sqrt{Mn} - 1$ competitive nodes.
- (2) Each H-node broadcasts at most D/M coded packets. The probability that a coded packet broadcast in a specific V-rectangle is at most

$$\frac{D}{M} \times \sqrt{\frac{M}{n}} = \frac{D}{\sqrt{nM}}.$$

- (3) Each broadcast generates $9M/10$ duplicate copies. Thus, from the Chernoff bound, we have that

$$\Pr(C_{(i,k)} \leq DM) \geq 1 - e^{-\frac{D}{300M}}$$

which implies that for sufficiently large n

$$\Pr(C_{(i,k)} \leq DM \forall k) \geq 1 - \frac{D}{M} e^{-\frac{D}{300M}} \geq 1 - e^{-\frac{D}{400M}}. \quad (74)$$

Let $\mathcal{T}_{(i,k)}$ denote the event that a B-duplicate packet is broadcast at time slot t in the transporting. If (i, k) is successfully duplicated, i.e., there are at least $4M/5$ B-duplicate copies of (i, k) , we have

$$\Pr(\mathcal{T}_{(i,k)}(t)) \geq \frac{4M}{5} \sqrt{\frac{M}{n}} \left(1 - \sqrt{\frac{M}{n}}\right)^{C_{(i,k)} + \frac{9M}{10} - 1}.$$

Further, let $\mathcal{T}_{(i,k)}$ denote the event that at least one copy of (i, k) is broadcast in the transporting. Then for sufficiently large n , we can obtain that

$$\begin{aligned} & \Pr(\mathcal{T}_{(i,k)} | C_{(i,k)} \leq DM) \\ & \geq 1 - \left(1 - \frac{4M}{5} \sqrt{\frac{M}{n}} \left(1 - \sqrt{\frac{M}{n}}\right)^{DM + 9M/10 - 1}\right)^D \\ & \geq \frac{1}{4}. \end{aligned}$$

Let C_i^b denote the number of distinct coded packets of node i broadcast in the transporting, i.e.,

$$C_i^b = \sum_{k=1}^{D/M} 1_{\mathcal{T}_{(i,k)}}.$$

Since different coded packets of node i are broadcast in different V-rectangles, $\{\mathcal{T}_{(i,k)}\}$ are mutually independent. From the Chernoff bound, we have

$$\Pr\left(C_i^b \geq \frac{D}{7M} \mid A_i \geq \frac{2D}{3M}, C_{(i,k)} \leq MD \forall k\right) \geq 1 - 2e^{-\frac{D}{900M}}. \quad (75)$$

In the transporting, a T-duplicate copy will be dropped if the node carrying it obtains another packet destined to the same destination. Consider a T-duplicate packet (i, k, l) carried by node l . Note following facts.

- (1) Coded packets of node i are broadcast in at most D/M V-rectangles.
- (2) Each rectangle contains at most $9M/10$ B-duplicate copies from node i .

Thus, the probability of (i, k, l) dropped at time slot t is at most

$$\frac{D}{M} \sqrt{\frac{M}{n}} \left(1 - \sqrt{\frac{M}{n}}\right)^{\frac{9M}{10}}.$$

The node mobility is independent across time, so the probability of (i, k, l) dropped in the transporting is at most

$$1 - \left(1 - \frac{D}{M} \sqrt{\frac{M}{n}} \left(1 - \sqrt{\frac{M}{n}}\right)^{\frac{9M}{10}}\right)^D \leq \frac{1}{M^2}.$$

Let \bar{N}_i^d denote the number of duplicate packets dropped in the transporting. Note that $9MC_i^b/10$ T-duplicate packets are generated, and each of them has probability $1/M^2$ to be dropped. Using the Markov inequality, we have

$$\Pr\left(\bar{N}_i^d \geq \frac{MC_i^b}{100}\right) \leq \frac{90}{M^2}$$

which implies

$$\Pr\left(C_i \geq \frac{9}{10} C_i^b\right) \geq 1 - \frac{90}{M^2} \quad (76)$$

since otherwise, more than $MC_i^b/100$ duplicate copies are dropped. Inequality (41) follows from inequalities (74)–(76).

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