# Coding Achieves the Optimal Delay-Throughput Trade-offs in Mobile Ad-Hoc Networks: A Hybrid Random Walk Model with Fast Mobiles

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***Abstract*— The delay-throughput trade-off of a mobile wireless network under the two-dimensional i.i.d mobility model has been investigated in [14], where we showed that the optimal trade-off can be achieved using rate-less codes. In this paper, we extend the result to a hybrid random walk model with fast mobiles. We first prove that the maximum throughput per source-destination (S-D) pair is $O(\sqrt{D/n})$ when $S = o(1)$ and $D = \omega(|\log S|/S^2)$, and then propose a joint coding-scheduling scheme to achieve the maximum throughput when $S = o(1)$ and $D$ is both $\omega(\max\{(\log^2 n)|\log S|/S^6, \sqrt[3]{n}\log n\})$ and $o(n/\log^2 n)$, where $n$ is the number of mobile nodes and $S$ is the step size of the hybrid random walk.**

## I. Notations

The following notations are used throughout this paper, given non-negative functions $f(n)$ and $g(n)$:

(1) $f(n) = O(g(n))$ means there exist positive constants $c$ and $m$ such $f(n) \leq cg(n)$ for all $n \geq m$.
(2) $f(n) = \Omega(g(n))$ means there exist positive constants $c$ and $m$ such that $f(n) \geq cg(n)$ for all $n \geq m$. Namely, $g(n) = O(f(n))$.
(3) $f(n) = \Theta(g(n))$ means that both $f(n) = \Omega(g(n))$ and $f(n) = O(g(n))$ hold.
(4) $f(n) = o(g(n))$ means that $\lim_{n\to\infty} f(n)/g(n) = 0$.
(5) $f(n) = \omega(g(n))$ means that $\lim_{n\to\infty} g(n)/f(n) = 0$. Namely, $g(n) = o(f(n))$.

## II. Introduction

The delay-throughput trade-offs in mobile ad-hoc networks have received much attention since the work of Grossglauser and Tse [6], where they showed that the throughput of ad-hoc networks can be significantly improved by exploring the node mobility. Recently the trade-off was investigated under different mobility models, which include the i.i.d. mobility [11], [13], [8], [14], one-dimensional mobility [1], [5], random walk [2], [3], [4], [12], hybrid random walk [12] and Brownian motion [9].

In [14], we demonstrated that the optimal trade-offs of the i.i.d. mobility models can be achieved using rate-less codes. In this paper, we extend the result to a hybrid random walk model which was introduced in [12]. We first prove that the maximum throughput per S-D pair is $O(\sqrt{D/n})$ when $S = o(1)$ and $D = \omega(|\log S|/S^2)$, and then propose a joint coding-scheduling scheme to achieve the maximum throughput when $S = o(1)$ and $D$ is both $\omega(\max\{(\log^2 n)|\log S|/S^6, \sqrt[3]{n}\log n\})$ and $o(n/\log^2 n)$. Note that the optimal delay-throughput trade-off are established under some conditions on $D$. When these conditions are not met, the trade-off is still unknown in general, though a trade-off has been established under an assumption regarding packet replication in [12]. We also would like to mention that when the step size of the hybrid random walk is $1/\sqrt{n}$, our hybrid random walk model is identical to the random walk model studied in [3], [4], where the optimal delay-throughput trade-off has been obtained.

The remainder of the paper is organized as follows: In Section III, we introduce the communication and mobility model. The joint coding-scheduling algorithm and the main results are presented in Section IV. Finally, we conclude our paper in Section V.

## III. Model

In this section, we first present the mobility and wireless interference models used in this paper. Then the definitions of delay and throughput are provided.

**Hybrid Random Walk Model:** Consider an ad-hoc network where wireless mobile nodes are positioned in a unit square. The unit square is divided into $1/S^2$ squares of equal size. Each of the smaller square will be called an RW-cell (random walk cell). The unit square is assumed to be a torus, i.e., the top and bottom edges are assumed to touch each other and similarly the left and right edges also are assumed to touch other. A node which is in one RW-cell at a time slot moves to one of its eight adjacent RW-cells or stays in the same RW-cell in the next time-slot with each move being equally likely as Figure 1. Two RW-cells are said to be adjacent if they share a common point. The node position within the RW-cell is randomly uniformly selected. There are $n$ S-D pairs in the network. Each node is both a source and a destination. Without loss of generality, we assume that the destination of node $i$ is node $i+1$, and the destination of node $n$ is node 1.

**Communication Model:** We assume the protocol model introduced in [7] in this paper. Let $\text{dist}(i, j)$ denote the Euclidean

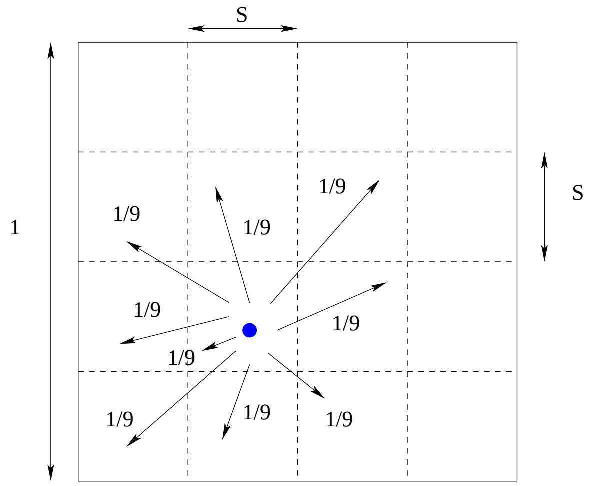


Fig. 1. Two-Dimensional Random Walk Model

distance between node $i$ and node $j$, and $r_i$ to denote the transmission radius of node $i$. A transmission from node $i$ can be successfully received at node $j$ if and only if following two conditions hold:

(i) $\text{dist}(i,j) \le r_i$;
(ii) $\text{dist}(k,j) \ge (1+\Delta)r_i$ for each node $k \ne i$ which transmits at the same time, where $\Delta$ is a protocol-specified guard-zone to prevent interference.

We further assume that at each time slot, at most $W$ bits can be transmitted in a successful transmission.

**Fast mobility:** The mobility of nodes is at the same time-scale as the data transmission, so $W$ is a constant independent of $n$ and only one-hop transmissions are feasible in single time slot.

**Delay and Throughput:** We consider hard delay constraints in this paper. Given a delay constraint $D$, a packet is said to be successfully delivered if the destination obtains the packet within $D$ time slots after it is sent out from the source.

Let $\Lambda_i[T]$ denote the number of bits successfully delivered to the destination of node $i$ in time interval $[0,T]$. A throughput of $\lambda$ per S-D pair is said to be feasible under the delay constraint $D$, if for any $\varepsilon > 0$, there exists $n_0(D,\varepsilon)$ such that for any $n \ge n_0(D,\varepsilon)$, there exists a coding/routing/scheduling algorithm with the property that each bit transmitted by a source is received at its destination with probability at least $1-\varepsilon$, and

$$\lim_{T\to\infty} \Pr\left(\frac{\Lambda_i[T]}{T} \ge \lambda,\ \forall\, i\right) = 1. \tag{1}$$

## IV. The Optimal Delay-Throughput Trade-off

### A. Upper Bound

First, we introduce following notations which will be used in our proof.

- $b$ : Index of a bit.
- $(i,b)$ : Bit $b$ generated by source $i$.
- $(i,b,j)$ : The copy of bit $b$ which originated from node $i$ and is carried by node $j$.

Suppose that $(i,b,j)$ is generated at the end of time slot $t_{(i,b,j)}$. We let $\tilde{L}_{(i,b,j)}$ denote the minimum distance between node $j$ and the destination of node $i$ from time slot $t_{(i,b,j)}+1$ to $t_{(i,b,j)}+D$, i.e.,

$$\tilde{L}_{(i,b,j)} = \min_{t_{(i,b,j)}+1 \le t \le t_{(i,b,j)}+D} \text{dist}(j, i+1)(t).$$

*Theorem 1:* Consider the hybrid random walk model with fast mobiles. Given $S = o(1)$ and a delay constraint $D = \omega(|\log S|/S^2)$, the maximum throughput per S-D pair satisfies

$$\lambda \le \frac{480\sqrt{2}W}{\Delta}\sqrt{\frac{D}{n}}.$$

*Proof:* Consider bit $(i,b,j)$, and let $N^{\text{rw}}_{(i,b,j)}$ denote the number of time slots, between $t_{(i,b,j)}+1$ and $t_{(i,b,j)}+D$, at which node $j$ and the destination of node $i$ are in the same RW-cell or neighboring RW-cells. Further define

$$L = \sqrt{\frac{\lambda}{72W\pi D}}.$$

Since $\lambda = O(1)$, $S = o(1)$, and $D = \omega(|\log S|/S^2)$, there exists $n_0$ such that for any $n \ge n_0$,

$$L = \sqrt{\frac{\lambda}{72W\pi D}} \le \frac{S}{\sqrt{|\log S|}} \le S,$$

which implies that for any $n \ge n_0$, the distance of two nodes is less than or equal to $L$ only if they are in the same RW-cell or neighboring RW-cells. Thus, we have

$$\begin{aligned}
&\Pr\left(\tilde{L}_{(i,b,j)} \le L\right)\\
=\ & \sum_{K=1}^{\infty} \Pr\left(\tilde{L}_{(i,b,j)} \le L | N^{\text{rw}}_{(i,b,j)} = K\right) \Pr\left(N^{\text{rw}}_{(i,b,j)} = K\right)\\
\le\ & \sum_{K=1}^{\infty} \left(1 - \left(1 - \frac{\pi L^2}{S^2}\right)^K\right) \Pr\left(N^{\text{rw}}_{(i,b,j)} = K\right)\\
=\ & 1 - E\left[\left(1 - \frac{\pi L^2}{S^2}\right)^{N^{\text{rw}}_{(i,b,j)}}\right]\\
\le\ & 1 - \left(1 - \frac{\pi L^2}{S^2}\right)^{E\left[N^{\text{rw}}_{(i,b,j)}\right]},
\end{aligned}$$

where the first inequality follows from the fact that the node position within a RW-cell is randomly uniformly selected, and the last inequality follows from the Jensen's inequality.

Next consider the mobility of node $j$ relative to node $i+1$, i.e., assume we observe node $j$ from node $i+1$ so node $i+1$ is stationary from our point of view. Then, the mobility can be regarded as a composition of two independent random walks: the movement of node $j$ and a random walk opposite to the movement of node $i+1$. So the mobility of node $j$ relative to node $i+1$ is also a random walk. Then from Lemma 5 provided in Appendix B, it is easy to see that

$$E\left[N^{\text{rw}}_{(i,b,j)}\right] \le \frac{99}{10} S^2 D$$

for $D = \omega(|\log S|/S^2)$, and we have

$$\Pr\left(\tilde{L}_{(i,b,j)} \le L\right) \le \left(1 - \left(1 - \frac{\pi L^2}{S^2}\right)\right)^{\frac{99}{10}S^2 D} \quad (2)$$
$$\le 36L^2 D. \quad (3)$$

The rest of the proof follows from the proofs of Lemma 2 and Theorem 3 in [14]. ■

### B. Joint Coding-Scheduling Algorithms

In this subsection, we propose a joint coding-scheduling scheme to achieve the maximum throughput obtained in Theorem 1. We first define four different types of packets.

- Source packets: Packets which have to be transmitted from source to destination.
- Coded packets: Packets generated by Raptor codes. We use $(i,k)$ to denote the $k^{\text{th}}$ coded packet of node $i$.
- Duplicate packets: Each coded packet could be broadcast to other nodes to generate multiple copies, called duplicate packets. We use $(i,k,j)$ to denote a copy of $(i,k)$ carried by node $j$.
- Deliverable packets: Duplicate packets that happen to be in the same cells as their destinations.

We divide the unit square into square cells with each side of length equal to $1/\sqrt[4]{nD}$. A cell is said to be a *good cell* at time $t$ if the number of nodes in the cell is more than $9M/10$ but no more than $11M/10$, where $M = \sqrt{n/D}$, the mean number of nodes in each cell.

**Joint Coding-Scheduling Scheme:** We group every $6D$ time slots into a super time slot. At each super time slot, the nodes transmit packets as follows, where $M = \sqrt{n/D}$.

(1) **Raptor Encoding:** Each source takes $2D/(25M)$ source packets, and uses Raptor codes to generate $D/M$ coded packets.

(2) **Broadcasting:** This step consists of $D$ time slots. At each time slot, the nodes do the following:

  (i) In each good cell, one node is randomly selected. If the selected node has not already transmitted all of its $D/M$ coded packets, then it broadcasts a coded packet that was not previously transmitted to $9M/10$ other nodes in the cell during the mini-slot allocated to that cell. Recall that our choice of packet size allows one node in every good cell to transmit during every time slot.

  (ii) All nodes check the duplicate packets they have. If more than one duplicate packet has the same destination, randomly keep one and drop the others.

(3) **Receiving:** This step consists of $5D$ time slots. At each time slot, if a cell contains no more than two deliverable packets, the deliverable packets are delivered to their destinations using one-hop transmissions during the mini-slot allocated to that cell. At the end of this step, all undelivered packets are dropped. The destinations decode the received coded packets using Raptor decoding.

In the next theorem, we show that the throughput obtained in Theorem 1 can be achieved using the algorithm above under some constraints on $D$.

*Theorem 2:* Consider the Joint Coding-Scheduling Algorithm. Suppose $S = o(1)$, $D$ is both $\omega(\max\{(\log^2 n)|\log S|/S^6, \sqrt[3]{n}\log n\})$ and $o(n/\log^2 n)$, and the delay constraint is $6D$. Then given any $\varepsilon > 0$, there exists $n_0$ such that for any $n \ge n_0$, every source packet sent out can be recovered at the destination with probability at least $1-\varepsilon$, and furthermore

$$\lim_{T\to\infty} \Pr\left(\frac{\Lambda_i[T]}{T} \ge \left(\frac{W}{2520}\right)\left(\sqrt{\frac{D}{n}}\right)\ \forall i\right) = 1. \quad (4)$$

*Proof:* We consider one super time slot which consists of $6D$ time slots, and calculate the probability that the $2D/(25M)$ source packets from node $i$ are fully recovered at the destination. Let $\mathscr{G}$ denote the event that the cells are all good in the whole super time slot. Since $D = o\left(n/\log^2 n\right)$ implies $M = \omega(\log n)$, we have

$$\Pr(\mathscr{G}) \ge 1 - \frac{1}{n^2} \quad (5)$$

by applying the Chernoff bound and union bound. A coded packet is said to be successfully duplicated if it has $4M/5$ duplicate copies after the broadcasting step. Let $A_i$ denote the number of coded packets of node $i$ which are successfully duplicated. We will first show that

$$\Pr\left(A_i \ge \frac{16D}{25M}\middle|\mathscr{G}\right) \ge 1 - \frac{55D}{n} - e^{-\frac{D}{600M}}. \quad (6)$$

Then let $B_i$ denote the number of distinct coded packets receieved at the destination, we will show that

$$\Pr\left(B_i \ge \frac{3}{25}\frac{D}{M}\middle|A_i \ge \frac{16}{25}\frac{D}{M}\right) \ge 1 - 2e^{-\frac{\log D}{8500}} - e^{-\frac{D}{500M\log D}}. \quad (7)$$

From inequalities (5), (6) and (7), we can conclude that under the Joint Coding-Scheduling Algorithm, at each super time slot, the $2D/(25M)$ source packets can be successfully recovered with a probability at least

$$1 - \frac{1}{n^2} - \frac{55D}{n} - e^{-\frac{D}{600M}} - 2e^{-\frac{\log D}{8500}} - e^{-\frac{D}{500M\log D}}.$$

The rest of the proof is the same as the proof of Theorem 4 in [14].

**Proof of inequality (6):** Consider the broadcasting step. Note that when $\mathscr{G}$ occurs, node $i$ is selected to broadcast with a probability at least $10/(11M)$ at each time slot. Let $\mathscr{B}_i[t]$ denote the event that node $i$ is selected to broadcast in time slot $t$. From the Chernoff bound, we have

$$\Pr\left(\sum_{t=1}^{D} 1_{\mathscr{B}_i[t]} \ge \frac{9}{11}\frac{D}{M}\middle|\mathscr{G}\right) \ge 1 - e^{-\frac{D}{600M}}. \quad (8)$$

So node $i$ broadcasts $9D/(11M)$ coded packets with a high probability. Note that each broadcast generates $9M/10$ duplicated copies, and a duplicate packet of node $i$ might be dropped if the node carrying it obtains another packet from node $i$. Next we calculate the number of duplicated copies dropped in step (ii), and then obtain the number of coded packets of node $i$ which are successfully duplicated. First we have that the probability that a duplicate copy dropped is at most

$$\frac{11}{10}\frac{DM}{n}\times\frac{10}{9}\frac{1}{M}=\frac{11}{9}\frac{D}{n}. \tag{9}$$

due to the following two facts:

(a) Let $\mathscr{H}_{ji}[D]$ denote the event that node $j$ is in the same cell as node $i$ in at least one of $D$ consecutive time slots. Similar to (3), it can be shown that when $D$ satisfies constraints given in the theorem, we have

$$\Pr\left(\mathscr{H}_{ji}[D]\right)\leq\frac{11}{10}\frac{DM}{n} \tag{10}$$

(b) When $\mathscr{G}$ occurs, node $i$ is selected to broadcast with a probability at most $10/(9M)$ at each time slot.

Now suppose source $i$ broadcasts $\tilde{D}_i$ coded packets, so $9M\tilde{D}_i/10$ duplicate copies are generated. Let $\tilde{N}_i^d$ denote the number of duplicate packets of node $i$ dropped in the broadcasting step. From the Markov inequality and inequality (9), we have

$$\Pr\left(\tilde{N}_i^d\geq\frac{M\tilde{D}_i}{50}\,\middle|\,\mathscr{G},\sum_{t=1}^{D}1_{\mathscr{B}_i[t]}=\tilde{D}_i\right)\leq\frac{55D}{n},$$

which implies

$$\Pr\left(A_i\geq\frac{4}{5}\tilde{D}_i\,\middle|\,\mathscr{G},\sum_{t=1}^{D}1_{\mathscr{B}_i[t]}=\tilde{D}_i\right)\geq 1-\frac{55D}{n} \tag{11}$$

since otherwise, the number of duplicate packets from node $i$ left in the network is less than $49M\tilde{D}_i/50$. Inequality (6) follows from inequalities (11) and (8).

**Proof of inequality (7):** We group every $3D/\log D$ time slots into big time slots, named b-time-slots and indexed by $t_b$, and then divided every b-time-slot into three equal parts, indexed by $t_{b,1}$, $t_{b,2}$ and $t_{b,3}$ as in Figure 2. We first calculate the

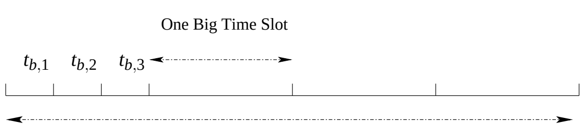


Fig. 2. The Division of A Super Time Slot

probability that coded packet $(i,k)$ is delivered in $t_{b,2}$. Let $\mathscr{H}_{(i,k)}[t_{b,2}]$ denote the event that at least one copy of packet $(i,k)$ becomes deliverable in some time slot belonging to $t_{b,2}$. If $(i,k)$ is a successfully duplicated packet, i.e., it has at least $4M/5$ duplicated copies, we have

$$\Pr\left(\mathscr{H}_{(i,k)}\left[t_{b,2}\right]\right)\geq 1-\left(1-\frac{S^2M}{n}\right)^{\frac{4M}{5}\frac{4D}{5S^2\log D}}-\frac{1}{n}\geq\frac{3}{5}\frac{1}{\log D} \tag{12}$$

due to the following two facts:

(a) Since $D/\log D=\omega(\log n|\log S|/S^6)$, from Lemma 5 provided in Appendix B, we know that with a probability at least $1-1/n$, two nodes are in the same RW-cell for at least $4D/(5S^2\log D)$ time slots in $t_{(b,2)}$.

(b) When two nodes are in the same RW-cell, the probability that they are in the same cell is $S^2M/n$.

Next note that the duration of $t_{b,1}$ and $t_{b,3}$ are at a larger order than the mixing time of the random walk (the mixing time is defined in Appendix B). From the definition of the mixing time, we have that at any time slot belonging to $t_{b,2}$, the nodes are almost uniformly distributed in the unit square. Let $\mathscr{D}_{(i,k)}[t_{b,2}]$ denote the event that coded packet $(i,k)$ is delivered to its destination in $t_{b,2}$. From inequality (12) and the argument used to prove inequality (13) in [14], we have

$$\Pr\left(\mathscr{D}_{(i,k)}[t_{b,2}]\right)\geq\frac{3}{20\log D}. \tag{13}$$

Now let $\mathbf{x}_t$ denote the positions of the nodes at time slot $t$, and

$$\tilde{\mathbf{X}}=\{\mathbf{x}_t\}_{t=\frac{3(k-1)D}{\log D}+1}$$

for $k=1,\ldots,5\log D/3$. Note that $\{\mathscr{D}_{(i,k)}[t_{b,2}]\}$ are mutually independent given $\tilde{\mathbf{X}}$. Further, let $\mathscr{D}_{(i,k)}$ denote the event that $(i,k)$ is delivered in the receiving step, from inequality (13), we have

$$\Pr\left(\mathscr{D}_{(i,k)}\middle|\tilde{\mathbf{X}}\right)\geq 1-\left(1-\frac{3}{20}\frac{1}{\log D}\right)^{5\log D/3}\geq\frac{1}{5}.$$

Thus we have that

$$E\left[B_i\,\middle|\,\tilde{\mathbf{X}},A_i\geq\frac{16}{25}\frac{D}{M}\right]\geq\frac{16}{125}\frac{D}{M}. \tag{14}$$

We next bound the number of distinct coded packets of node $i$ becoming deliverable in one b-time-slot. Similar to inequality (12), it can be shown that

$$\Pr\left(\mathscr{H}_{(i,k)}\left[t_b\right]\right)\leq\frac{3}{\log D}.$$

Since node $i$ broadcasts at most $D/M$ coded packets and duplicate copies of node $i$ are carried by different mobiles, we have

$$\Pr\left(\sum_{k=1}^{D/M}1_{\mathscr{H}_{(i,k)}[t_b]}\leq\frac{16}{5}\frac{D}{M\log D}\right)\geq 1-e^{-\frac{D}{400M\log D}}.$$

Let $\tilde{\mathscr{F}}_i$ denote the event that node $i$ obtains no more than $16D/(5M\log D)$ coded packets at each b-time-slot in the receiving step. From the union bound, we have for sufficiently large $n$,

$$\Pr\left(\tilde{\mathscr{F}}_i\right)\geq 1-\left(\frac{5}{3}\log D\right)\left(e^{-\frac{D}{400M\log D}}\right)\geq 1-e^{-\frac{D}{500M\log D}}. \tag{15}$$

Finally let $\mathbf{X}_{t_b}$ denote an $n\times(3D/\log D)$ matrix where the $(i,t)$ entry is the position of node $i$ at the $t^{\text{th}}$ time slot of b-time-slot $t_b$, and $B_i(\tilde{\mathbf{X}},A_i,\mathscr{F}_i)$ denote the number of distinct coded packets delivered to the destination of node $i$ given $(\tilde{\mathbf{X}},A_i,\mathscr{F}_i)$. It is easy to see that $\{\mathbf{X}_{t_b}\}$ are mutually independent given

$(\tilde{\mathbf{X}}, A_i, \tilde{\mathscr{F}}_i)$, and $B_i(\tilde{\mathbf{X}}, A_i, \mathscr{F}_i)$ is a function of $\{\mathbf{X}_{t_b}\}$, i.e., there exits a function $f_{(\tilde{\mathbf{X}}, A_i, \mathscr{F}_i)}$ such that

$$B_i(\tilde{\mathbf{X}}, A_i, \mathscr{F}_i) = f_{(\tilde{\mathbf{X}}, A_i, \mathscr{F}_i)}\left(\mathbf{X}_1, \ldots, \mathbf{X}_{5\log D/3}\right).$$

Furthermore, the function $f_{(\tilde{\mathbf{X}}, A_i, \mathscr{F}_i)}$ satisfies the following condition,

$$\Big| f_{(\tilde{\mathbf{X}}, A_i, \mathscr{F}_i)}\left(\mathbf{X}_1, \ldots, \mathbf{X}_{t_b-1}, \mathbf{X}_{t_b}, \mathbf{X}_{t_b+1}, \ldots, \mathbf{X}_{5\log D/3}\right) - f_{(\tilde{\mathbf{X}}, A_i, \mathscr{F}_i)}\left(\mathbf{X}_1, \ldots, \mathbf{X}_{t_b-1}, \mathbf{Y}_{t_b}, \mathbf{X}_{t_b+1}, \ldots, \mathbf{X}_{5\log D/3}\right) \Big| \leq \frac{16}{5}\frac{D}{M\log D}.$$

So from Lemma 3 provided in Appendix A, we can conclude that

$$\Pr\left(B_i(\tilde{\mathbf{X}}, A_i, \mathscr{F}_i) \geq E\left[B_i \left| \tilde{\mathbf{X}}, A_i, \tilde{\mathscr{F}}_i\right.\right] - \frac{1}{125}\frac{D}{M}\right) \geq 1 - 2e^{-\frac{\log D}{8500}} \quad (16)$$

for any $\tilde{\mathbf{X}}$ and $A_i$, and inequality (7) follows from inequalities (14), (15) and (16). ■

## V. Conclusion

In this paper, we investigated the delay-throughput trade-off in ad-hoc networks under a hybrid random walk model with fast mobiles, and showed that that the optimal delay-throughput trade-off can be achieved using a joint coding-scheduling scheme under some constraints on $D$.

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## Appendix A: Azuma-Hoeffding inequality

*Lemma 3:* Suppose that $X_0, \ldots, X_n$ are independent random variables, and there exists a constant $c > 0$ such that $f(\mathbf{X}) = f(X_1, \ldots, X_n)$ satisfies the following condition for any $i$ and any set of values $x_1, \ldots, x_n$ and $y_i$ :

$$|f(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n) - f(x_1, \ldots, x_{i-1}, y_i, x_{i+1}, \ldots, x_n)| \leq c.$$

Then we have

$$\Pr\left(|f(\mathbf{X}) - E[f(\mathbf{X})]| \geq \delta\right) \leq 2e^{-\frac{-2\delta^2}{nc^2}}.$$

*Proof:* A detailed proof can be found in [10]. ■

## Appendix B: Two-Dimensional Random Walk

Consider a two-dimensional random walk on a unit torus with $1/\tilde{S}^2$ points. A node moves to one of eight neighboring points or stays in the same points from one time slot to another as in Figure 3. We introduce following definitions.

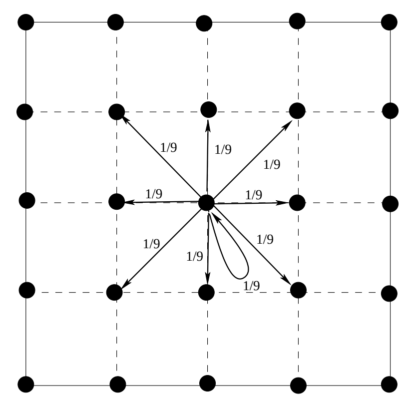


Fig. 3. Two Dimensional Random Walk

- Transition matrix $\mathbf{P}$ : $\mathbf{P} = \left[P_{\mathbf{i},\mathbf{j}}\right]$ where $P_{\mathbf{i},\mathbf{j}}$ is the probability of moving from point $\mathbf{i}$ to point $\mathbf{j}$.
- Stationary distribution $\Pi$ : A vector which satisfies the equation $\Pi\mathbf{P} = \Pi$.
- Hitting time $T_h(\mathbf{i}, \mathbf{j})$ : Time taken for a node to move from point $\mathbf{i}$ to point $\mathbf{j}$.
- Mixing time $T_m$ :

$$T_m = \inf_t \sup_{\mathbf{i}} \sum_{\mathbf{j}} \left|P^t_{\mathbf{ij}} - \Pi_{\mathbf{j}}\right| \leq \tilde{S}^4,$$

where $P^t_{\mathbf{ij}}$ is the $(\mathbf{i}, \mathbf{j})^{\text{th}}$ entry of $\mathbf{P}^t$.

*Lemma 4:* Consider the two-dimensional random walk, we have

$$E[T_h(\mathbf{i}, \mathbf{j})] = O(|\log \tilde{S}|/\tilde{S}^2)$$

for any $\mathbf{i}$ and $\mathbf{j}$, and

$$T_m = O(|\log \tilde{S}|/\tilde{S}^2).$$

*Proof:* The proof of the hitting time result is presented Lemma 14 of [3]. The mixing time result holds since the two-dimensional random walk can be decomposed into two independent one-dimensional random walks, and the mixing time of a one-dimensional random walk on a circle with $1/\tilde{S}$ points is $O(|\log\tilde{S}|/\tilde{S}^2)$. ■

*Lemma 5:* Let $N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D]$ denote the number of visits to point $\mathbf{j}$ in $D$ time slots starting from point $\mathbf{i}$ and ending at point $\mathbf{k}$. If $D=\omega(|\log\tilde{S}|/\tilde{S}^2)$, we have

$$\frac{9}{10}D\tilde{S}^2 \le E[N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D]] \le \frac{11}{10}D\tilde{S}^2. \tag{17}$$

If $D=\omega(\alpha|\log\tilde{S}|/\tilde{S}^6)$, we further have

$$\Pr\left(\frac{4}{5}D\tilde{S}^2 \le N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D] \le \frac{6}{5}D\tilde{S}^2\right) \ge 1-2e^{-\frac{\alpha}{625}}. \tag{18}$$

*Proof:* First we have

$$T_h(\mathbf{i},\mathbf{j}) + \sum_{l=1}^{N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D]-1} T_h^l(\mathbf{j},\mathbf{j}) + T_h(\mathbf{j},\mathbf{k}) = D,$$

where $T_h^l(\mathbf{j},\mathbf{j})$ is the time duration between $l^{\text{th}}$ visits to point $\mathbf{j}$ and $(l+1)^{\text{th}}$ visits to point $\mathbf{j}$. Taking the expectation on both sides, we have

$$E[T_h(\mathbf{i},\mathbf{j})]+E[N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D]]E[T_h(\mathbf{j},\mathbf{j})]-E[T_h(\mathbf{j},\mathbf{j})]+E[T_h(\mathbf{j},\mathbf{k})]=D,$$

which implies

$$E[N_{\mathbf{i},\mathbf{j},\mathbf{k}}[D]] = \frac{D-E[T_h(\mathbf{i},\mathbf{j})]-E[T_h(\mathbf{j},\mathbf{k})]+E[T_h(\mathbf{j},\mathbf{j})]}{E[T_h(\mathbf{j},\mathbf{j})]}.$$

Inequality (17) follows from the facts that $E[T_h(\mathbf{i},\mathbf{j})] = O(|\log\tilde{S}|/\tilde{S}^2)$ and $E[T_h(\mathbf{j},\mathbf{j})]=1/\tilde{S}^2$.

Next let $\mathbf{x}[t]$ denote the position of the node at time slot $t$, $\tilde{\mathbf{X}}$ denote $\{\mathbf{x}[t]\}$ for $t=1,\frac{D\tilde{S}^4}{\alpha}+1,\frac{2D\tilde{S}^4}{\alpha}+1,\dots,D-\frac{D\tilde{S}^4}{\alpha},D$, and $\mathbf{X}_m$ denote $\{\mathbf{x}[t]\}$ for $t=\frac{mD\tilde{S}^4}{\alpha}+1,\frac{mD\tilde{S}^4}{\alpha},\dots,\min\left\{D,\frac{(m+1)D\tilde{S}^4}{\alpha}\right\}$ where $m=0,\dots,\alpha/\tilde{S}^4-1$. Further, let $N_{\mathbf{j},\tilde{\mathbf{X}}}[D]$ denote the number of visits to point $\mathbf{j}$ given $\tilde{\mathbf{X}}$. It is easy to see that for any $\tilde{\mathbf{X}}$ there exits a function $f_{\tilde{\mathbf{X}}}$ such that

$$N_{\mathbf{j},\tilde{\mathbf{X}}}[D] = f_{\tilde{\mathbf{X}}}\left(\mathbf{X}_1,\dots,\mathbf{X}_{\alpha/\tilde{S}^4-1}\right),$$

where $\{\mathbf{X}_m\}$ are mutually independent given $\tilde{\mathbf{X}}$. Note that $\mathbf{X}_m$ contains the position information from time slot $mD\tilde{S}^4/\alpha+1$ to time slot $(m+1)D\tilde{S}^4/\alpha$, so

$$\left| f_{\tilde{\mathbf{X}}}\left(\mathbf{X}_0,\dots,\mathbf{X}_{m-1},\mathbf{X}_m,\mathbf{X}_{m+1},\dots,\mathbf{X}_{\alpha/\tilde{S}^4-1}\right) - f_{\tilde{\mathbf{X}}}\left(\mathbf{X}_0,\dots,\mathbf{X}_{m-1},\mathbf{Y}_m,\mathbf{X}_{m+1},\dots,\mathbf{X}_{\alpha/\tilde{S}^4-1}\right)\right| \le \frac{D\tilde{S}^4}{\alpha}.$$

Next note that $D\tilde{S}^4/\alpha=\omega(|\log\tilde{S}|/\tilde{S}^2)$ when $D=\omega(|\log\tilde{S}|/\tilde{S}^6)$, so from inequality (17), we can conclude that for any $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$,

$$\frac{9}{10}\frac{D\tilde{S}^6}{\alpha} \le E\left[N_{\mathbf{i},\mathbf{j},\mathbf{k}}\left[\frac{D\tilde{S}^4}{\alpha}\right]\right] \le \frac{11}{10}\frac{D\tilde{S}^6}{\alpha},$$

which implies that

$$\frac{9}{10}D\tilde{S}^2 \le E\left[N_{\mathbf{j},\tilde{\mathbf{X}}}[D]\right] \le \frac{11}{10}D\tilde{S}^2$$

holds for any $\tilde{\mathbf{X}}$. Then from the Azuma-Hoeffding inequality (Lemma 3), we have that

$$\Pr\left(\left|N_{\mathbf{j},\tilde{\mathbf{X}}}[D]-E[N_{\mathbf{j},\tilde{\mathbf{X}}}[D]]\right| \le \frac{9}{100}D\tilde{S}^2\right) \ge 1-2e^{-\frac{\alpha}{625}},$$

for any $\tilde{\mathbf{X}}$, and inequality (18) holds. ■