

# On Efficient Data Transport with Mobile Carriers

Jung Ryu, Lei Ying and Sanjay Shakkottai

**Abstract**—In this paper, we consider a network of stationary nodes that rely on mobile nodes to transport data between them. We assume the mobile nodes can control their mobility pattern to respond to traffic loads, as well as satisfy some other secondary objectives, such as surveillance requirements. We study this problem in the framework of cost minimization, where we derive a dual iterative algorithm that results in optimal mobility pattern for minimizing network wide cost. We then implement our proposed algorithm and evaluate its performance on a testbed.

**Index Terms**—Mobility control, routing, mobile carriers, disruption tolerant networking, network optimization.

## I. INTRODUCTION

DATA delivery across “disconnected clusters” of nodes using mobile nodes are of increasing interest. Applications include those in Disruption Tolerant Networking [1], battlefield networks, and more generally in scenarios where there is a lack of infrastructure. Mobile nodes potentially would serve multiple functions, e.g., surveillance and monitoring of the region, along with supporting data delivery. Further, in many applications, there is likely to be some flexibility in choosing the trajectories of these mobile nodes (i.e., controllable mobility).

For concreteness, consider an exploration outpost in a remote corner of the world. At such a location, it would be difficult to establish infrastructure for traditional cellular or WiFi networks due to cost, availability of power sources, etc. Relying on satellites can be expensive and would only support low data rates. At such remote locations, one can utilize a group of reconnaissance mobiles (such as UAVs) to transport data from one part of the network to another. These UAVs can be used to patrol the premise periodically in order to ensure security, and they can be readily equipped with radio transceivers to pick-up and drop-off data at different locations as they patrol, thus serving a dual purpose.

Because these UAVs play a critical role in providing connectivity, there has been a surge of interest in developing reliable and efficient algorithms for these types of networks that use mobile data carriers. However, due to the opportunistic and intermittent nature of the mobile connections (the wireless connections are formed and broken as the mobiles move about) and high link delays, the traditional routing and rate control algorithms, such as OSPF and TCP, used in the Internet suffer performance degradation if used in highly intermittent and opportunistic environment.

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J. Ryu and S. Shakkottai are with the Department of Electrical and Computer Engineering, The University of Texas at Austin (e-mail: {jung.ryu, shakkott}@mail.utexas.edu).

L. Ying is with the Department of Electrical and Computer Engineering at Iowa State University in Ames, IA (e-mail: leiying@iastate.edu).

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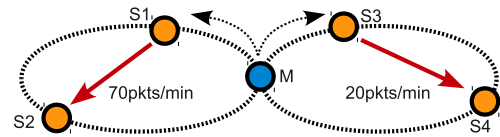


Fig. 1. A simple network with two modes: The mobile can choose to orbit the left route or the right route. If it chooses the left (right) route, the mobile will come into contact with stationaries S1 and S2 (S3 and S4). On contact, the mobile can drop-off and pick-up data to and from the stationaries. In this figure, S1 generates a stream of data for S2 (S3 for S4). To serve the flow S1-S2, the mobile has to go into the left route, pick up data from S1 and drop them off to S2. If the flow S1-S2 has higher rate than S3-S4, the mobile should go into left route more often than right route.

One way to mitigate the problem of opportunistic and random connectivity is through controlled mobility. By controlling the motion of mobile data carriers, one can make the connections less opportunistic and random, and more periodic or more predictable. There is a vast number of literature available on controlled mobility, ranging from robotics to operations research [2], [3]. Extensive study has been done on problems such as minimizing the travel cost subject to some constraints and finding an optimal routes for pick up and drop off of goods [4].

In this paper, we focus on minimum cost dynamic routing with controlled mobility. Specifically, we study a network of stationary nodes that rely on mobiles to transport data between them, and the data rates are not known and may vary over time. The cost for data transport consists of two parts:

*First*, there is a per-packet per-route cost – this reflects the cost of transmitting a packet over a specific route. For instance, such linear costs have been used in literature [5] to minimize hop count. This cost could be source dependent (e.g., a hard-to-reach source might be penalized with a higher cost). In our study, we allow any source and mobile dependent per-packet cost to reflect this.

*Second*, there is a per-route cost – a mobile is allowed to periodically change trajectories, and the cost is a function of the trajectory that is chosen. For instance, longer trajectories (that potentially use more fuel) could be penalized with a higher cost (in our model, we allow any positive cost per trajectory).

In this paper, we design an algorithm that will 1) *guarantee throughput optimality* and 2) *minimize the sum cost over the entire network*. We do this by enabling the mobiles to control their own routes of operation in response to the traffic demand. Without controlled mobility, one would have to resort to fixing the routes a priori, and this could lead to an unstable network, as we demonstrate in the next section.

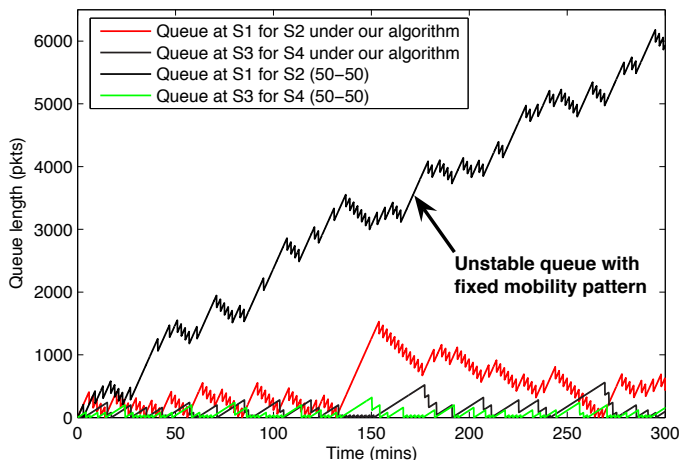


Fig. 2. If the route operations percentages are set to 50-50 a priori, the source rates we have in Figure 1 can not be supported and will lead to unstable queues. However, as long as the source rates are in the capacity region (which takes into account the patrol requirements), our algorithm will support them and stabilize the queues.

## II. ILLUSTRATIVE EXAMPLE

Consider the simple network shown in Figure 1. We have one mobile that can choose from two “routes.” In the left route, it will come into contact with stationary nodes S1 and S2 (in that order); in the right route, with S3 and S4. On contact, the mobile can drop off and pick up data to and from the stationary nodes. Each route requires two minutes to finish, and after one route is finished, the mobile returns to the center and can choose the next route. On each contact, 200 packets can be picked up or dropped off by the mobile. In addition to transporting data between nodes, the mobile also has a purpose of patrolling the area, and must travel each route at least 15% of the time.

S1 generates data destined to S2 at a rate of 70pkts/min, and S3 generates data destined to S4 at a rate of 20pkts/min. The mobile does not know the data rates, and the rates can vary over time. If the mobility pattern of the mobile is fixed, then the network may not be able to support the two traffic flows. For example, if the mobile patrols each route 50% of the time, then the flow from S1 to S2 cannot be supported in the network and the queues are unstable, as shown in Figure 2.

In this paper, we develop an algorithm that controls the mobility pattern of the mobile dynamically so that it not only satisfies the surveillance requirement *but also* stabilizes the queues whenever it is possible. In the simple example in Figure 1, using our algorithm, the mobile may patrol the left route 75% of the time and the right route 25% so that each route is patrolled at least 15% of the time (surveillance requirement), and both traffic flows are supported and all queues are stable.

## III. RELATED WORKS

The networks that utilize mobile carriers to transport data have been studied extensively recently by [1], [6]–[17] and others. The primary focus of [10], [13], [16], [17] is to increase the data delivery probability and reduce delivery

latency through *replication* in the context of *delay-tolerant networks* (DTN). Replication is useful in networks where mobile carriers move randomly because it increases the opportunities to transfer data from mobile nodes to static nodes and vice versa. In networks where the mobility patterns of mobiles are fixed, replication is not necessary. However, the drawback of fixed mobility patterns is that the network cannot dynamically respond to the changes in the traffic loads. In addition, as long as data is delivered to the destination, it is considered sufficient, but in networks where the mobility pattern can be controlled, we can not only guarantee data delivery, but also the most efficient and optimal network resource utilization.

An extensive simulation study of a network where the mobile messengers are used to transport data among clusters has been done in [9]; there, the impact of different mobility patterns of the messengers on delay and efficiency is examined. In [18] and [19], algorithms that control the mobility of a mobile data collector in a sensor network to reduce data collection delay have been developed. Both papers explore the trade-off between mobility and wireless transmissions energy. In [20] a trajectory control algorithm such that the mobile data collector dynamically switches its trajectory to be closer to sensor nodes with more data to transmit in order to save TX power has been developed. In addition, [21], [22] explore sensor networks where nodes can reconfigure their positions dynamically in order to enhance coverage and life span. The work in [23] is a study of a routing protocol based on controlled mobility that minimizes the distance traveled by mobile carriers to reach the destination. However, the mobile messengers in [9], [18]–[21], [23] *cannot adapt their mobility patterns to traffic loads*, the result of which will be unstable queues as illustrated in the previous section. Experimental evaluation of heuristic mobility control algorithm that can respond to changing network capacity and demand is presented in [24]. Optimization based approaches for mobile data collection have been studied in [38], where the authors study the problem of mobile sinks that need to collect data from various sources before their respective buffers overflow. The algorithm based on using the deadlines (time to fill buffers at various nodes) is shown to be NP-complete, and various heuristics are then explored to alleviate this. Further, in [25], the authors study the problem of transporting data to a single collector from a collection of stationary data generators via reinforcement learning techniques.

The optimization framework based on back-pressure [26] that is used in our paper has been used extensively in [5], [27]–[33], [39] and many others for developing efficient resource allocation schemes in wired and wireless networks in the context of congestion control and back-pressure routing and rate control. The networks studied in these papers consist of static nodes and the links are not intermittent, while in this paper, we focus on intermittently connected networks, and develop an algorithm that controls mobility to support network traffic flows while guaranteeing some other objectives such as surveillance requirement. The optimization algorithm that is most similar to ours is the one developed in [39]. The focus of [39] however is on an abstract problem of optimizing stochastic renewal systems; ours focuses on minimizing the cost of transporting data over mobile networks.

In this paper, we combine the optimization framework used for back-pressure routing with mobility control in order to develop a dynamic *throughput and cost optimal* mobility control algorithm that allows multiple mobiles to transport data among a collection of stationary nodes. Our contributions include:

- 1) We formulate a cost minimization framework for the network where the mobile carrier adapts its mobility pattern to support traffic flows among stationary nodes while satisfying a secondary surveillance objective. We present the min-cost mobility control algorithm that is throughput and cost optimal, and then develop a practical distributed algorithm.
- 2) We implement our practical distributed algorithm and present experimental results on our Pharos test bed [34] using the Click router [35], where we implement the radio and network aspects and emulate mobility.

#### IV. NETWORK MODEL

The network consists of  $L$  stationary nodes and one mobile carrier<sup>1</sup>. The stationary nodes do not move and can not communicate with each other directly; they must rely on the mobile carrier to transport data among them. We assume that stationary nodes generate data for other stationary nodes. Let  $d_l$  denote the destination node of the data stream generated by stationary node  $l$ , and let  $x_l^{d_l}$  (pkts/time slot) be the corresponding average rate. Let  $\mathbf{x} = \{x_l^{d_l}\}$ . Define

$$1_{\{l,u\}} = \begin{cases} 1 & \text{if } l' = d_l \\ 0 & \text{else.} \end{cases}$$

A stationary node can exchange data with the mobile carrier when the two come into contact. During each contact, the mobile carrier can send  $\eta_d$  packets to the stationary node, and receive  $\eta_p$  packets from the stationary node. We call the transmissions from a stationary node to the mobile carrier a *pick up*, and the transmissions from the mobile carrier to a stationary node a *drop off*.

We assume that there is a terminal  $V$  in the network. This terminal does not generate data. It is there to facilitate notation and understanding of the definition of route, which we define next.

**Definition 1:** A **route** of the mobile carrier starts and ends at the terminal. The route is a (finite) set of tuples  $(s, n_s)$  where  $s$  is a stationary node and  $n_s$  is the number of times the mobile visits  $s$  on that route. The route is further specified by the time required to patrol that route. ■

**Assumption 1:** We assume that there are  $J$  routes for the mobile carrier, which are indexed by  $j$ . Stationary node  $l$  is assessed a cost  $a_{l,j}$  for every packet picked up by the mobile to be sent over route  $j$  ( $a_{l,j}$  is called *pick up cost*). The mobile incurs a cost of  $b_j$  per time slot when patrolling route  $j$ . ■

An example of a route is

$$R_1 = \{(V, 2), (l_1, 1), (l_2, 4), 10mins\}.$$

<sup>1</sup>We assume one mobile carrier only to simplify the notations. This assumption however can be easily removed, see section V-B for the multiple mobile formulation.

On this route, the mobile starts at  $V$ , visits  $l_1$  once and  $l_2$  four times before returning to  $V$ . The time the mobile takes to patrol  $R_1$  is 10mins. The route  $R_1$  is different from the route  $R_2 = \{(V, 2), (l_1, 2), (l_2, 4), 10mins\}$  because  $l_1$  is visited twice on  $R_2$  but only once on  $R_1$ .  $R_1$  is also different from the route  $R_3 = \{(V, 2), (l_1, 1), (l_2, 4), 5mins\}$  because it takes less time to patrol  $R_3$ . Note that the mobile carrier must return to the terminal before switching onto another route. Though not shown, the terminal in the network in Figure 1 would be located where the left and right routes meet (right under where the mobile is).

We can associate higher  $b_j$  with the routes on which the mobile moves faster since that would require more fuel. We assume that  $a_{l,j} \leq a_{\max}$ ,  $\forall l, j$  and  $b_j \leq b_{\max}$ ,  $\forall j$ . We let  $f_j$  denote the fraction of time the mobile carrier is on route  $j$ , and  $T_j$  denote the time required to patrol route  $j$  (in units of time slots). Assume that  $T_{\min} \leq T_j \leq T_{\max}$ ,  $\forall j$ . If  $N$  routes have been patrolled, and out of the  $N$  routes, the mobile patrolled route  $j$   $N_j$  times, then  $f_j = \frac{N_j T_j}{\sum_{j'} N_{j'} T_{j'}}$ . Note that not all stationary nodes may be included on one route, so the mobile may have to switch from one route to another to transport packets from a source to its destination.

Definition 1 specifies a route of a mobile via the the collection of stationary nodes that a mobile visits, the number of times that each of them is visited, and the total time taken to physically traverse this route. Note that several physical paths (i.e., the actual geographic paths) can share the same mobile route as specified by this description (e.g., the difference between two physical paths could be the order in which the stationary nodes are visited, or that the actual trajectory could be different; however, the path characteristics are summarized by Definition 1 could be the same). In this case, multiple physical paths would be mapped to the same route. The reason for our choice of these parameters to define a route is that the list of stationary nodes along with the number of times that they are visited describe the transfer capacity between the mobile and stationary nodes, and this along with the time duration of the mobile route is needed to describe the rate of data transfer between the mobile and stationary nodes (rate = (number of contacts) × (packets transferred per contact) / (time duration of mobile route), see Table I). Different physical paths with the same route parameters in Definition 1 lead to the same data transfer constraints, hence we do not distinguish between them (as the rest of the physical route properties are not relevant to our model for data transfer), and our algorithm will treat them all as the same route.

Note that for some reason, if we do need to distinguish between physical paths that have the same route (e.g., with different costs), it is easy to do so by one of two means: (i) simply change the original route time durations ( $T$ , which is originally the same for the two routes) to be  $(T - \epsilon)$  and  $(T + \epsilon)$ , for some arbitrarily small  $\epsilon$ ; or (ii) augment the notation in Definition 1 to have an additional route-index parameter (that distinguishes between the two physical paths). All that this will change is to add an extra route queue in the Min-Cost Mobility Control Algorithm (see Section V), and the algorithm will proceed to load-balance between these two routes based on the costs.

**Surveillance Requirement:** The mobile must periodically

TABLE I  
DESCRIPTION OF VARIABLES USED

| Notation        | Description  |
|-----------------|--|
| $x_l^{d_l}$     | rate stationary $l$ generates data for stationary $d_l$                |
| $y_{l,j}^{d_l}$ | rate mobile picks up data from $l$ on route $j$ , destined for $d_l$   |
| $p_j (f_j)$     | minimum (actual) fraction of time mobile spends on route $j$           |
| $T_j$           | time duration of mobile route $j$                                      |
| $a_{l,j}$       | cost $l$ pays to have a packet picked up on route $j$                  |
| $b_j$           | cost per unit time mobile pays to spend on route $j$                   |
| $q_{l,j}$       | queue $l$ maintains for packets to be picked on route $j$              |
| $Q_l$           | queue mobile maintains for packets destined for $l$                    |
| $\delta_{l,j}$  | fraction of total resource on route $j$ stationary $l$ uses            |
| $w_j$           | counter mobile uses to satisfy the surveillance req. (3)               |
| $K$             | tuning param. that controls optimality and queue sizes                 |
| $\kappa$        | tuning parameter that controls the size of the counter $w_j$           |
| $\eta_P$        | # of packets picked up per contact                                     |
| $\eta_D$        | # of packets dropped off per contact                                   |
| $\zeta_{l,j}$   | # of contacts mobile makes with $l$ on route $j$                       |
| $P_{l,j}$       | data pick-up rate from $l$ on route $j$ ( $\eta_P \zeta_{l,j} / T_j$ ) |
| $D_{l,j}$       | data drop-off rate to $l$ on route $j$ ( $\eta_D \zeta_{l,j} / T_j$ )  |

patrol route  $j$  to guarantee that  $f_j \geq p_j$  for some  $p_j \geq 0$ ,  $\forall j$ . ■

We let  $\zeta_{l,j}$  denote the number of contacts that can be made between the mobile carrier and stationary node  $l$  on route  $j$ . Let  $P_{l,j}$  and  $D_{l,j}$  denote the rates that the mobile can receive from and transmit to node  $l$  on route  $j$ , i.e.,  $P_{l,j} = \frac{\eta_P \zeta_{l,j}}{T_j}$  and  $D_{l,j} = \frac{\eta_D \zeta_{l,j}}{T_j}$ .

Each stationary node  $l$  maintains a queue  $q_{l,j}$  for each route  $j$ ,  $j = 1, \dots, J$ . Node  $l$  deposits into queue  $q_{l,j}$  the packets it wants the mobile to pick up on route  $j$ . The mobile maintains a queue  $Q_{l'}$  for each destination node  $l'$ ,  $l' = 1, \dots, L$ .

**Definition 2:** We say the network is **stable** if all queues are bounded. ■

We let

$$1_{\{P_{l,j} > 0\}} = \begin{cases} 1 & P_{l,j} > 0 \\ 0 & \text{else.} \end{cases}$$

Finally, we define

$$\eta_{\max} = \max \left\{ \max_{l,j} \{P_{l,j} T_j\}, \max_{l,j} \{D_{l,j} T_j\} \right\}.$$

## V. MIN-COST MOBILITY CONTROL

In this section, we will introduce our min-cost mobility control algorithm. The variables associated with our algorithm along with their brief physical description are given in Table I. Define  $y_{l,j}^{d_l}$  to be the (average, long-term) rate at which data generated by stationary node  $l$  is picked up by the mobile carrier when patrolling route  $j$ . The objective of the min-cost mobility control is to find  $\mathbf{f} = \{f_j, j = 1, \dots, J\}$  and  $\mathbf{y} = \{y_{l,j}^{d_l}, l = 1, \dots, N, j = 1, \dots, J\}$  (pick up rate splitting) to (i) stabilize the network, i.e., guarantee that the queue lengths are bounded, (ii) minimize the average cost  $\sum_{l,j} y_{l,j}^{d_l} a_{l,j} + \sum_j b_j f_j$ , and (iii) satisfy the surveillance requirement. These decisions are determined adaptively via a dual-decomposition inspired mobility control algorithm.

**Definition 3:** We say the arrival rate  $\mathbf{x}$  is **supportable** if there exists  $(\mathbf{f}, \mathbf{y})$  such that:

$$\sum_j y_{l,j}^{d_l} = x_l^{d_l} \quad (1)$$

$$\sum_j f_j \leq 1 \quad (2)$$

$$f_j \geq p_j \in [0, 1], j = 1, \dots, J \quad (3)$$

$$y_{l,j}^{d_l} = \delta_{l,j} f_j P_{l,j}, \forall l, j \quad (4)$$

$$\sum_{j,l} 1_{\{l,\nu\}} \delta_{l,j} f_j P_{l,j} \leq \sum_j f_j D_{\nu,j}, \forall l' \quad (5)$$

$$0 \leq \delta_{l,j} \leq 1. \quad (6)$$

Let  $\Lambda$  be the set of all supportable arrival rates. Given  $\mathbf{x} \in \Lambda$ , let  $\Gamma_{\mathbf{x}}$  denote the set of  $(\mathbf{f}, \mathbf{y})$  that satisfy the equations in definition 3. An arrival rate  $\mathbf{x}$  is said to be supportable if there exist  $\mathbf{f}$  and  $\mathbf{y}$  such that: (i) The summation of  $f_j$  (the fraction of time a mobile spends on route  $j$ ) be less than one (eq. (2)) and the surveillance requirement (eq. (3)) is satisfied; (ii) The rate at which packets are transmitted from stationary node  $l$  to the mobile while it is patrolling route  $j$  is upper bounded by the pick up rate by the mobile on route  $j$  (eq. (4)); (iii) The rate at which the data for destination node  $l'$  are picked up from all stationary nodes and on all routes is limited by the combined rate at which data is dropped off to node  $l'$  on on all routes (eq. (5)). The parameter  $\delta_{l,j}$  controls the fraction of active contacts between node  $l$  and the mobile on route  $j$ ; when the mobile and stationary node  $l$  come into contact, they have the option of using only part, if any, of the transfer capacity of the contact; and (iv) The total rate the data are picked up from node  $l$  on all routes should be equal to the rate at which node  $l$  generates data (eq. (1)).

Note that we can not replace conditions (4) and (5) with

$$y_{l,j}^{d_l} \leq f_j P_{l,j}, \quad (7)$$

$$\sum_{j,l} 1_{\{l,\nu\}} f_j P_{l,j} \leq \sum_j f_j D_{\nu,j}. \quad (8)$$

Consider two sources  $s_1$  and  $s_2$ , with destinations  $d_1$  and  $d_2$ , respectively. Assume that  $s_1$  and  $s_2$  can be visited by the mobile only when the mobile is on route  $R_0$ , and  $x_{s_1}^{d_1} > x_{s_2}^{d_2}$ . Suppose the pick-up rates from  $s_1$  and  $s_2$  are the same ( $P_{s_1,R_0} = P_{s_2,R_0}$ ). Then if we enforce constraint (7), then  $x_{s_1}^{d_1} = y_{s_1,R_0}^{d_1} \leq f_{R_0} P_{s_1,R_0}$  and  $x_{s_2}^{d_2} = y_{s_2,R_0}^{d_2} \leq f_{R_0} P_{s_2,R_0} = f_{R_0} P_{s_1,R_0}$ . Assume that  $d_1$  and  $d_2$  are contacted by the mobile only when it is on routes  $R_1$  and  $R_2$ , respectively, and they are receiving data only from  $s_1$  and  $s_2$ , respectively. In addition, suppose that the drop-off rates to  $d_1$  and  $d_2$  are the same ( $D_{d_1,R_1} = D_{d_2,R_2}$ ). If we force constraint (8), then  $f_{R_0} P_{s_1,R_0} \leq f_{R_1} D_{d_1,R_1}$  and  $f_{R_0} P_{s_1,R_0} = f_{R_0} P_{s_2,R_0} \leq f_{R_2} D_{d_2,R_2}$  which implies that the mobile has to spend just as much time on  $R_2$  as  $R_1$ , even though  $x_{s_1}^{d_1} > x_{s_2}^{d_2}$ . We resolve this issue by using  $\delta_{l,j}$ .

The objective of the minimum-cost mobility control is to solve the following optimization problem: Given  $\mathbf{x} \in \Lambda$  and



for any fixed  $K > 0$ ,

$$\begin{aligned} & \text{minimize } \left\{ K \sum_{l,j} y_{l,j}^{d_l} a_{l,j} + K \sum_j f_j b_j \right\} \\ & \text{subject to } (\mathbf{f}, \mathbf{y}) \in \Gamma_{\mathbf{x}}. \end{aligned} \quad (9)$$

Let  $(\hat{\mathbf{f}}, \hat{\mathbf{y}})$  denote an optimal solution to (9).

As we will see later, by suitably choosing the value of parameter  $K > 0$  in the optimization problem, the solution generated by our algorithm will be sufficiently ‘‘close’’ to the optimal. Next, note that multiplying both sides of constraint (3) by any positive constant  $\kappa > 0$  does not change the condition. Thus, the partial Lagrange dual of the optimization problem (9) is the following<sup>2</sup>:

$$\begin{aligned} L(q_{l,j}, Q_{l'}, w_j) = & \\ & \min_{y_{l,j}^{d_l}, f_j, \delta_{l,j}} \left\{ K \sum_{l,j} y_{l,j}^{d_l} a_{l,j} - \sum_{l,j} q_{l,j} (\delta_{l,j} f_j P_{l,j} - y_{l,j}^{d_l}) \right. \\ & - \sum_{l'} Q_{l'} \left( \sum_j f_j D_{l',j} - \sum_{j,l} 1_{\{l,l'\}} \delta_{l,j} f_j P_{l,j} \right) \\ & \left. + K \sum_j f_j b_j - \sum_j \kappa w_j (f_j - p_j) \right\} \end{aligned}$$

subject to 1)  $\sum_j y_{l,j}^{d_l} = x_l^{d_l}$ , 2)  $\sum_j f_j \leq 1$ , and 3)  $\delta_{l,j} \in [0, 1]$ . As we will see later, the parameter  $\kappa$  is useful in our algorithm in order to match the time-scale of route selection with the time-scale of queue-length variation.

We now observe that we can decompose the Lagrange dual into two subproblems:

$$\begin{aligned} & \min_{y_{l,j}^{d_l} \geq 0} K \sum_j y_{l,j}^{d_l} a_{l,j} + \sum_j q_{l,j} y_{l,j}^{d_l} \\ & \text{s.t. } \sum_j y_{l,j}^{d_l} = x_l^{d_l} \end{aligned} \quad (10)$$

for each  $l = 1, \dots, L$  and

$$\begin{aligned} & \max_{f_j \geq 0, \delta_{l,j}} \sum_{l,j} q_{l,j} \delta_{l,j} f_j P_{l,j} + \sum_j \kappa w_j (f_j - p_j) \\ & + \sum_{l',j} Q_{l'} f_j \left( D_{l',j} - \sum_l 1_{\{l,l'\}} \delta_{l,j} P_{l,j} \right) \\ & - K \sum_j f_j b_j \\ & \text{s.t. } \sum_j f_j \leq 1 \text{ and } \delta_{l,j} \in [0, 1] \forall l, j \end{aligned} \quad (11)$$

Motivated by the dual decomposition, we propose the following min-cost mobility control algorithm. Here, the index  $k$  denotes the  $k$ -th route selection. If the  $k$ -th route is  $j$ , the time duration between  $k$ -th and  $(k+1)$ -th route selections is  $T_j$ . In the min-cost mobility control algorithm, a stationary node deposits its packets into a queue that solves the subproblem (10), and a mobile station selects its route by

solving subproblem (11). In addition to packet queues, each mobile station maintains a deficit counter for each route. The length of a deficit counter indicates the amount of times the mobile station needs to further patrol the route to fulfill the surveillance requirement.

### Min-Cost Mobility Control Algorithm

- (i) Stationary node  $l$  deposits  $y_{l,j}^{d_l}(k)$  packets into queue  $q_{l,j}$ , where

$$y_{l,j}^{d_l}(k) = \begin{cases} x_l^{d_l} & \text{if } j = j_l^*(k) \\ 0 & \text{else,} \end{cases} \quad (12)$$

and  $j_l^*(k) = \arg \min_j \{K a_{l,j} + q_{l,j}(k)\}$ .

- (ii) The  $k$ -th route  $j^*(k)$  selected by the mobile carrier is such that

$$\begin{aligned} j^*(k) \in \arg \max_j & \left\{ \sum_l q_{l,j}(k) \delta_{l,j}(k) P_{l,j} - K b_j \right. \\ & + \sum_{l'} Q_{l'}(k) \left( D_{l',j} - \sum_l 1_{\{l,l'\}} \delta_{l,j}(k) P_{l,j} \right) \\ & \left. + \kappa w_j(k) (1 - p_j) \right\} \end{aligned} \quad (13)$$

where

$$\delta_{l,j}(k) = \begin{cases} 1 & \text{if } 1_{\{P_{l,j} > 0\}} [q_{l,j}(k) - \sum_{l'} 1_{\{l,l'\}} Q_{l'}(k)] > 0 \\ 0 & \text{else.} \end{cases} \quad (14)$$

The mobile will pick up data from  $l$  on route  $j^*(k)$  if and only if  $\delta_{l,j^*(k)}(k) = 1$ . In addition,

$$f_j(k) = \begin{cases} 1 & \text{if } j = j^*(k) \\ 0 & \text{else,} \end{cases} \quad (15)$$

and

$$T(k) = T_{j^*(k)}. \quad (16)$$

- (iii) The queues are updated as follows:

$$\begin{aligned} q_{l,j}(k+1) = & [q_{l,j}(k) + \\ & T(k) (y_{l,j}^{d_l}(k) - \delta_{l,j}(k) P_{l,j} 1_{\{f_j(k)=1\}})]^+ \end{aligned} \quad (17)$$

$$\begin{aligned} Q_{l'}(k+1) = & \left[ Q_{l'}(k) + \sum_j 1_{\{f_j(k)=1\}} T(k) \right. \\ & \left. \times \left[ \sum_l 1_{\{l,l'\}} \delta_{l,j}(k) P_{l,j} - D_{l',j} \right] \right]^+ \end{aligned} \quad (18)$$

- (iv) The algorithm maintains a deficit counter  $w_j(k)$  for route  $j$  such that at step  $k$ ,  $w_j(k)$  is increases by  $T(k)p_j$  and decreases by  $T(k)f_j(k)$ , i.e.,

$$w_j(k+1) = [w_j(k) + T(k)p_j - 1_{\{f_j(k)=1\}} T(k)]^+. \quad (19)$$

The next two theorems demonstrate the stability and optimality of our proposed algorithm, respectively.

*Theorem 1:* Given  $\mathbf{x}$  such that  $(1+\epsilon)\mathbf{x} \in \Lambda$  for some  $\epsilon > 0$ , under the iterative algorithm above, the network is stable.

<sup>2</sup>In the notations,  $l'$  generally refers to the destination stationary, while  $l$  generally refers to the source stationary. If we want to be explicit, the destination of the flow originating from the source  $l$  is denoted  $d_l$ .

*Proof:* See Appendix A. ■

**Theorem 2:** Given  $\mathbf{x}$  such that  $(1+\epsilon)\mathbf{x} \in \Lambda$  for some  $\epsilon > 0$ , let  $\{\hat{f}_j, \hat{y}_{l,j}^{d_l}\}$  be the solution to the optimization problem (9). As  $K \rightarrow \infty$ , under our proposed algorithm,

$$\sum_{l,j} a_{l,j} \hat{y}_{l,j}^{d_l} + \sum_j \hat{f}_j b_j = \lim_{K \rightarrow \infty} \lim_{k' \rightarrow \infty} \left[ \frac{1}{\sum_{k \leq k'} T(k)} \times \left( \sum_{k \leq k'} \left( \sum_{l,j} T(k) a_{l,j} y_{l,j}^{d_l}(k) + \sum_j f_j(k) T(k) b_j \right) \right) \right]. \quad (20)$$

*Proof:* See Appendix B. ■

Note that  $\lim_{k' \rightarrow \infty} \frac{1}{\sum_{k \leq k'} T(k)} \sum_{k \leq k'} T(k) y_{l,j}^{d_l}(k)$  in equation (20) is the time-average data rate that the mobile picks up from stationary node  $l$  on route  $j$ , and  $\lim_{k' \rightarrow \infty} \frac{1}{\sum_{k \leq k'} T(k)} \sum_{k \leq k'} T(k) f_j(k)$  is the fraction of the time the mobile patrols route  $j$ .

### A. Impact of $K$ and $\kappa$

We know from [33] that  $(\mathbf{f}, \mathbf{y})$  obtained by our algorithm is within a factor of  $O(1/K)$  from  $(\hat{\mathbf{f}}, \hat{\mathbf{y}})$ , and while  $q_{l,j}$ ,  $Q_{l'}$  and  $\kappa w_j$  are  $O(K \max\{\eta_{\max}, \kappa T_{\max}\})$ , where  $\max\{\eta_{\max}, \kappa T_{\max}\}$  is the maximum amount by which  $q_{l,j}$ ,  $Q_{l'}$ , and  $\kappa w_j$  can increase between any two consecutive route selections. In order to keep  $q_{l,j}$  and  $Q_{l'}$  small and make the mobile go into “surveillance” route without having  $w_j$  build up, we choose  $\kappa$  such that  $\eta_{\max} \approx \kappa T_{\max}$ , i.e.  $\kappa = \Theta(\eta_{\max}/T_{\max})$ .

### B. Multiple Mobiles

So far, our model assumes only one mobile node to keep notation simple. However, one can easily extend the algorithm to the network with multiple mobiles. A straight forward way to extend our model is to have the stationary nodes maintain a queue for each route *and* each mobile, i.e. stationary node  $l$  maintains a queue  $q_{l,j,m}$  for route  $j$  for mobile  $m$ . Let  $f_{m,j}$  denote the frequency that mobile  $m$  operates on route  $j$ . We then have the constraint  $\sum_{j \in M_m} f_{m,j} \leq 1$  for each mobile  $m$ , where  $M_m$  is the set of routes mobile  $m$  can patrol. Each mobile also has its own “surveillance” or secondary objective constraints on  $f_{m,j}$ . Finally, each mobile solves the optimization problem (11) independently, without any cooperation with other mobiles.

It is easy to show that the results in this paper immediately carry over to this more general setting (the proofs are analogous to those presented here). Lastly, we note that this more general formulation supports multiple terminals, so that different mobiles can return to different terminal after each patrol.

## VI. PRACTICAL ALGORITHM

The min-cost mobility control algorithm we discussed in the previous section has two shortcomings. One is that the source nodes have to synchronize their queue selections with the mobile’s selection. The second issue is that the mobile has to know  $q_{l,j}$ ’s to make the route selection. However, we can take advantage of the secondary surveillance objective of the

mobile. While the mobile makes its surveillance round, it can collect the most up-to-date queue information while picking up and dropping off data, and use the information in selecting the next route.

Let  $k$  denote the  $k^{\text{th}}$  route the mobile operates on. We can recast the update equations (12), (13) and (14) into the following practical, distributed decision controls.

**Stationary node:** The first time the mobile contacts stationary node  $l$ , it communicates the values of  $a_{l,j}$ ’s corresponding only to the routes on which it will visit stationary node  $l$ . For each new packet, the stationary node  $l$  computes

$$j_l^* = \arg \min_j K a_{l,j} + q_{l,j} \quad (21)$$

and deposits it into  $q_{l,j_l^*}$ .

**Mobile:** Each time the mobile meets stationary node  $l$ , it collects all queue size information from node  $l$  at the end of the contact. At the end of the execution of a route, the mobile computes the next route  $f_j, \delta_{l,j} \in \{0, 1\}$  by maximizing

$$\sum_{l \in j} \mathbf{q}_{l,j} \delta_{l,j} f_j P_{l,j} + \sum_{l' \in j} Q_{l'} f_j (D_{l',j} - \delta_{l',j} P_{l',j}) + \sum_j \kappa w_j (f_j - p_j) - K \sum_j f_j b_j \quad (22)$$

where  $\mathbf{q}_{l,j}$  denotes the most up-to-date information known to the mobile.

## VII. EXPERIMENTAL RESULTS

We implemented the practical version of our algorithm as described in Section VI in Click [35] on our testbed [34] (see also [37] for more details). The purpose of the experiment is to demonstrate our practical algorithm can achieve the optimal value and to show that we can come arbitrarily close to the optimal value at the expense of longer queues as  $K \rightarrow \infty$ . The algorithm presented in Section V assumes perfect knowledge of the queue length by the mobile, while the practical version does not. To this end, we build our experimental network with WiFi cards on the Proteus platform [34]. We emulate the mobility by timed contacts, i.e., the mobile node is, in our emulation, stationary, and would make contact with one static node, then wait for some time before making contact with another static node. These timed contacts are implemented simply by turning on and off the appropriate wireless interfaces for the various nodes.

### A. Experiment 1

The network we used in this experiment is shown in Figure 1. We use the following routes:

$$\begin{aligned} R_1 &= \{(V, 2), (S1, 1), (S2, 1), 1min\} \\ R_2 &= \{(V, 2), (S1, 1), (S2, 1), 2mins\} \\ R_3 &= \{(V, 2), (S3, 1), (S4, 1), 1min\} \\ R_4 &= \{(V, 2), (S3, 1), (S4, 1), 2mins\} \end{aligned}$$

We used two flows, one from S1 to S2, at the rate of 40pkts/min and another one from S2 to S3 at the rate of 30pkts/min. The mobile and the nodes can transmit 100pkts per contact. The routes  $R_2$  and  $R_4$  must be patrolled at least 10% of the time, i.e.,  $p_2 = p_4 = 0.1$  and  $p_1 = p_3 = 0$ .

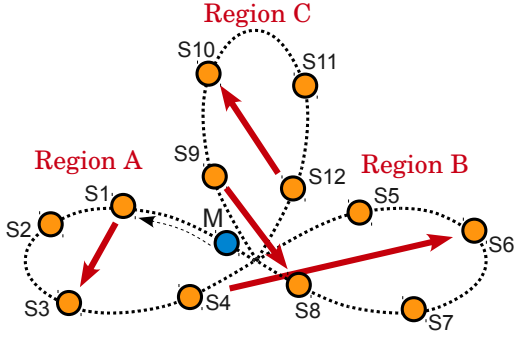


Fig. 3. A network with one mobile and 12 stationaries, 4 in each region. The mobile has two modes per region: a fast and a slow route. In fast route, the mobile goes through a region and comes back to the center in one minute; in slow route, it takes two minutes. All stationaries in the region are contacted in each route. (The terminal would be located where the three regions meet.)

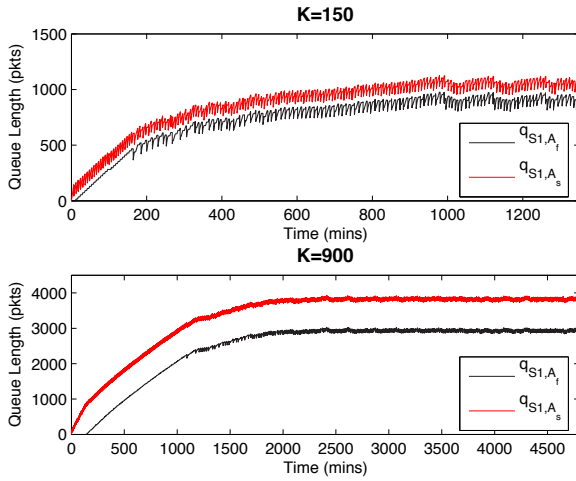


Fig. 4. Queue lengths observed at S1: Increasing  $K$  results in more optimal rate allocations, but it comes at the price of longer queues and longer time for convergence.

$a_{S1,R_1} = a_{S2,R_1} = K$  and  $a_{S1,R_2} = a_{S2,R_2} = 0$ ;  $b_{R_1} = b_{R_3} = K$  and  $b_{R_2} = b_{R_4} = 0$ . The rate splitting over the modes  $x_{S1,0}^{S2}$  and  $x_{S1,1}^{S2}$  is shown in Table II. (The optimal values in Tables II and III are obtained by numerically solving the optimization problem (9) using MATLAB.)

### B. Experiment 2

In this experiment, we used the network shown in Figure 3. The network is composed of three regions, A, B, and C. In each region, the mobile has two routes, fast and slow. We use  $A_f$  and  $A_s$  to denote the fast and slow routes in region A, respectively. ( $B_f$  and  $B_s$  for region B and  $C_f$  and  $C_s$  for region C.) On a fast route, the mobile goes through the region in one minutes; on slow route, the mobile takes two minutes. Each stationary node in a region is contacted once on each route made through that region.

On each route, the mobile makes contacts starting from the lowest numbered stationary to the highest. Each slow route is required to be patrolled at least 10% of the time, i.e.,  $p_{A_s} = p_{B_s} = p_{C_s} = 0.1$ . On each contact, the mobile can pick up and drop off 100pkts (200pkts total).

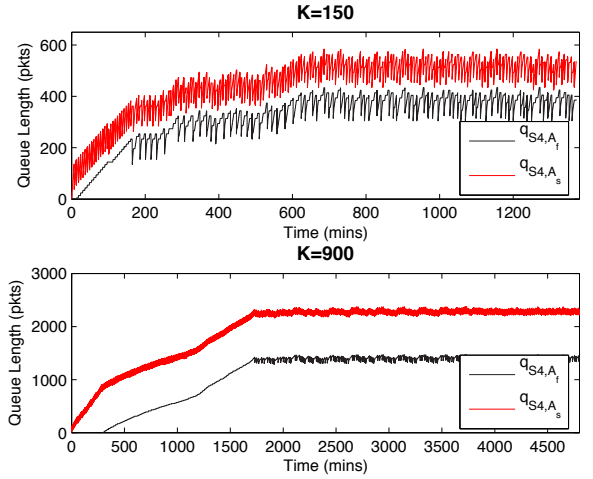


Fig. 5. Queue lengths observed at S4

TABLE II

EXPERIMENT 1: THE MOBILE HAS TO TRAVEL THE LEFT ROUTES  $R_1$  AND  $R_2$  TO PICK UP DATA FROM S2 AND TRAVEL THE RIGHT ROUTES  $R_3$  AND  $R_4$  TO DROP OFF DATA TO S3. THE MOBILE MUST PATROL ROUTES  $R_2$  AND  $R_4$  AT LEAST 10% OF THE TIME TO SATISFY THE SURVEILLANCE REQUIREMENT. THE STATIONARY NODES AND THE MOBILE WILL TRY TO UTILIZE THE ‘‘CHEAPER’’ ROUTES  $R_2$  AND  $R_4$  (THUS, MORE PACKETS ARE PICKED UP ON THOSE ROUTES) BEFORE THE MORE COSTLY ROUTES  $R_1$  AND  $R_3$ . (THE UNITS ARE PKTS/MIN.)

|                   | $K$  |        |       |         |
|-------------------|------|--------|-------|---------|
|                   | 150  | 300    | 600   | Optimal |
| $y_{S1,R_1}^{S2}$ | 20   | 18.376 | 15.8  | 15      |
| $y_{S1,R_2}^{S2}$ | 20   | 21.624 | 24.2  | 25      |
| $y_{S2,R_1}^{S3}$ | 9.9  | 8.598  | 5.97  | 5       |
| $y_{S2,R_2}^{S3}$ | 20.1 | 21.402 | 24.03 | 25      |

S1 generates data for S3 at rate 23pkts/min; S4 generates data for S6 at 20pkts/min. S9 generates data for S8 at rate 20pkts/min, and S12 generates data for S10 at rate 23pkts/min. The packet pick up costs  $a_{l,A_s}$ ,  $a_{l,B_s}$ , and  $a_{l,C_s}$  for the slow route are 0 for all stationary nodes; the pick up costs  $a_{l,A_f}$ ,  $a_{l,B_f}$  and  $a_{l,C_f}$  for the fast route are  $K$ . The route costs are  $b_{A_s} = b_{B_s} = b_{C_s} = 0$  and  $b_{A_f} = b_{B_f} = b_{C_f} = K$ . We let  $K = 150, 450, \text{ and } 900$ .  $\kappa$  is set to 100.

We compute the optimal rate splitting by solving the optimization problem (9) (shown in the ‘‘Optimal’’ column) and compare against the observed rate splitting under different values of  $K$ . As predicted by Theorem 2, the rate allocation approaches the optimal allocation as  $K$  increases as shown in Table III. The price of being close to the optimal rates is long queues, as demonstrated in Figures 4 and 5 for  $K = 150$  and 900.

## VIII. CONCLUSION

In this paper, we studied a network that uses a mobile carrier to transport data between stationary nodes. In our work, the mobile can change its movement dynamically to respond to the data traffic in the network. We have developed a cost minimization framework for such network and developed a joint mobile-stationary algorithm that minimizes the sum cost, and demonstrated our algorithm on a wireless testbed.

TABLE III  
EXPERIMENT 2: AS  $K$  INCREASES, THE RATE SPLITTING APPROACHES  
THE OPTIMAL. (THE UNITS ARE PKTS/MIN.)

|                     | $K$    |        |        |         |
|---------------------|--------|--------|--------|---------|
| $y_{l,j}^{d_l}$     | 150    | 450    | 900    | Optimal |
| $y_{S1,A_f}^{S3}$   | 10.488 | 9.5    | 8.51   | 8.5     |
| $y_{S1,A_s}^{S3}$   | 12.512 | 13.5   | 14.49  | 14.5    |
| $y_{S4,A_f}^{S6}$   | 7.6    | 6.172  | 5.4    | 5.5     |
| $y_{S4,A_s}^{S6}$   | 12.4   | 13.828 | 14.6   | 14.5    |
| $y_{S9,C_f}^{S8}$   | 7.64   | 6.112  | 5.378  | 5.5     |
| $y_{S9,C_s}^{S8}$   | 12.36  | 13.888 | 14.622 | 14.5    |
| $y_{S12,C_f}^{S10}$ | 10.78  | 9.33   | 8.372  | 8.5     |
| $y_{S12,C_s}^{S10}$ | 12.22  | 13.67  | 14.628 | 14.5    |

## IX. ACKNOWLEDGMENTS

The authors would like to express their gratitude to Prof. Christine Julien and Prof. Sriram Vishwanath for permitting the use of the Pharos testbed [34], [36] to generate the experimental results presented in this paper. We acknowledge the support of NSF Grant CNS-0721380 and CNS-0953165, the DARPA ITMANET program, and the DTRA grants HDTRA1-08-1-0016 and HDTRA1-09-1-0055

### APPENDIX A: PROOF OF THEOREM 1

Consider the Lyapunov function  $V(k) = \sum_{l,j} (q_{l,j}(k))^2 + \sum_{l'} (Q_{l'}(k))^2 + \kappa \sum_j (w_j(k))^2$ . Define  $\Delta V(k) = V(k+1) - V(k)$ . Note that

$$\begin{aligned} \Delta V(k) \leq & \sum_{l,j} (q_{l,j}(k+1) - q_{l,j}(k)) (2q_{l,j}(k) + \eta_{\max}) \\ & + \sum_{l'} (Q_{l'}(k+1) - Q_{l'}(k)) (2Q_{l'}(k) + \eta_{\max}) \\ & + \kappa \sum_j (w_j(k+1) - w_j(k)) (2w_j(k) + T_{\max}) \end{aligned} \quad (23)$$

since  $(q_{l,j}(k+1))^2 - (q_{l,j}(k))^2 = (q_{l,j}(k+1) + q_{l,j}(k)) (q_{l,j}(k+1) - q_{l,j}(k))$  and  $(q_{l,j}(k+1) + q_{l,j}(k)) \leq 2q_{l,j}(k) + \eta_{\max}$ . (Likewise for  $Q_{l'}(k)$  and  $w_j(k)$ .) The maximum amount by which  $q_{l,j}(k)$  and  $Q_{l'}(k)$  can increase in one iteration is  $\eta_{\max}$ ; the maximum amount by which  $w_j(k)$  can increase in one iteration is  $T_{\max}$ .

We prove that there exists  $q_{\max}$  such that if  $q_{l,j}(k)$  or  $Q_{l'}(k) > q_{\max}$  for some  $l, j, l'$ , and  $k$ , then

$$\Delta V(k) < -\alpha \quad (24)$$

where  $\alpha > 0$ . Equation (24) would prove the evolution of  $\{q_{l,j}(k), Q_{l'}(k)\}$  is upper bounded.

Define:

$$\tilde{q}_{l,j}(k) = \max\{T(k)\delta_{l,j}(k)f_j(k)P_{l,j} - q_{l,j}(k), 0\} \quad (25)$$

$$\tilde{Q}_{l'}(k) = \max\left\{T(k)\sum_j f_j(k)D_{l',j} - Q_{l'}(k), 0\right\} \quad (26)$$

$$\tilde{w}_j(k) = \max\{T(k) - w_j(k), 0\}. \quad (27)$$

Note that if  $q_{l,j}(k) \geq \eta_{\max}$  and  $Q_{l'}(k) \geq \eta_{\max}$ , then  $\tilde{q}_{l,j}(k) = 0$  and  $\tilde{Q}_{l'}(k) = 0$ , respectively, and if  $w_j(k) \geq T_{\max}$ , then  $\tilde{w}_j(k) = 0$ .

Using equations (17), (18), (19) (25), (26), and (27), the Lyapunov down drift equation (23) can be bounded as

$$\begin{aligned} \Delta V(k) \leq & 2 \sum_{l,j} \left[ T(k) \left( y_{l,j}^{d_l}(k) - \delta_{l,j}(k) f_j(k) P_{l,j} \right) + \tilde{q}_{l,j}(k) \right] q_{l,j}(k) \\ & + 2 \sum_{l'} \left[ f_j(k) T(k) \left( \sum_l 1_{\{l,l'\}} \delta_{l,j}(k) P_{l,j} - D_{l',j} \right) \right. \\ & \left. + \tilde{Q}_{l'}(k) \right] Q_{l'}(k) \\ & + 2\kappa \sum_j (T(k)p_j - T(k)f_j(k) + \tilde{w}_j(k)) w_j(k) \\ & + 2 \sum_{l,j} \eta_{\max}^2 + 2 \sum_{l'} \eta_{\max}^2 + 2\kappa \sum_j T_{\max}^2 \end{aligned}$$

because 1)  $q_{l,j}(k+1) - q_{l,j}(k) \leq 2\eta_{\max}$ , 2)  $Q_{l'}(k+1) - Q_{l'}(k) \leq 2\eta_{\max}$ , 3)  $w_j(k+1) - w_j(k) \leq 2T_{\max}$ , and 4)

$$\begin{aligned} q_{l,j}(k+1) - q_{l,j}(k) \leq & T(k) \left( y_{l,j}^{d_l}(k) - \delta_{l,j}(k) f_j(k) P_{l,j} \right) + \tilde{q}_{l,j}(k), \end{aligned}$$

$$Q_{l'}(k+1) - Q_{l'}(k) \leq$$

$$f_j(k) T(k) \left( \sum_l 1_{\{l,l'\}} \delta_{l,j}(k) P_{l,j} - D_{l',j} \right) + \tilde{Q}_{l'}(k),$$

and  $w_j(k+1) - w_j(k) \leq T(k)(p_j - f_j(k)) + \tilde{w}_j(k)$ .

Because of equations (25), (26) and (27), we have

$$\tilde{q}_{l,j}(k) q_{l,j}(k) \leq \eta_{\max}^2 \quad (28)$$

$$\tilde{Q}_{l'}(k) Q_{l'}(k) \leq \eta_{\max}^2 \quad (29)$$

$$\tilde{w}_j(k) w_j(k) \leq T_{\max}^2. \quad (30)$$

Let

$$A(k) = \sum_{l,j} y_{l,j}^{d_l}(k) q_{l,j}(k) \quad (31)$$

and

$$\begin{aligned} B(k) = & \sum_{l,j} \delta_{l,j}(k) f_j(k) P_{l,j} q_{l,j}(k) \\ & + \sum_{l',j} f_j(k) Q_{l'}(k) \left( D_{l',j} - \sum_l 1_{\{l,l'\}} \delta_{l,j}(k) P_{l,j} \right). \end{aligned} \quad (32)$$

Let

$$C = 2 \sum_{l,j} \eta_{\max}^2 + 2 \sum_{l'} \eta_{\max}^2 + 2\kappa \sum_j T_{\max}^2. \quad (33)$$

Then,

$$\begin{aligned} \Delta V(k) \leq & 2A(k)T(k) - 2B(k)T(k) + 2C \\ & - 2\kappa \sum_j T(k)(f_j(k) - p_j)w_j(k) \end{aligned} \quad (34)$$

due to equations (28), (29), (30), (31), (32), and (33).



Since  $(1 + \epsilon)\mathbf{x} \in \Lambda$ , there exist  $\{\tilde{y}_{l,j}^{d_l}\}$ ,  $\{\tilde{f}_j\}$ , and  $\{\tilde{\delta}_{l,j}\}$  in  $\Gamma_{(1+\epsilon)\mathbf{x}}$  that satisfy the definition 3. Let

$$\begin{aligned} \tilde{B}(k) &= \sum_{l,j} \tilde{\delta}_{l,j} \tilde{f}_j P_{l,j} q_{l,j}(k) \\ &+ \sum_{l',j} \tilde{f}_j Q_{l'}(k) \left( D_{l',j} - \sum_l 1_{\{l,l'\}} \tilde{\delta}_{l,j} P_{l,j} \right) \end{aligned} \quad (35)$$

Then,

$$\begin{aligned} \Delta V(k) &\leq 2T(k) \left[ A(k) - \left( B(k) - \tilde{B}(k) + \tilde{B}(k) \right) \right. \\ &\quad \left. - \kappa \sum_j (f_j(k) - p_j) w_j(k) \right] + 2C. \end{aligned}$$

To the RHS of the above inequality, we add and subtract the following two terms 1)  $2K \sum_j f_j(k) b_j T(k)$  2)  $2K \sum_j \tilde{f}_j b_j T(k)$  and add  $2\kappa T(k) \sum_j (\tilde{f}_j - p_j) w_j(k)$  ( $\geq 0$  since  $\tilde{f}_j$  satisfies inequality (3)). Thus, we get

$$\begin{aligned} \Delta V(k) &\leq 2T(k) \left[ A(k) - \left( B(k) - \tilde{B}(k) + \tilde{B}(k) \right) \right. \\ &\quad \left. - K \sum_j f_j(k) b_j + K \sum_j \tilde{f}_j b_j \right) - K \sum_j f_j(k) b_j \\ &\quad + K \sum_j \tilde{f}_j b_j - \kappa \sum_j (f_j(k) - p_j) w_j(k) \\ &\quad \left. + \kappa \sum_j (\tilde{f}_j - p_j) w_j(k) \right] + 2C \\ &\leq 2A(k)T(k) - 2\tilde{B}(k)T(k) + 2C \\ &\quad - 2K \sum_j f_j(k) b_j T(k) + 2K \sum_j \tilde{f}_j b_j T(k). \end{aligned} \quad (36)$$

Inequality (36) holds because

$$\begin{aligned} B(k) - K \sum_j f_j(k) b_j + \kappa \sum_j (f_j(k) - p_j) w_j(k) \\ \geq \tilde{B}(k) - K \sum_j \tilde{f}_j b_j + \kappa \sum_j (\tilde{f}_j - p_j) w_j(k) \end{aligned}$$

by algorithms (13) and (14). Hence,

$$\begin{aligned} \Delta V(k) &\leq 2A(k)T(k) - 2\tilde{B}(k)T(k) + 2C \\ &\quad - 2KT(k) \sum_j b_j (f_j(k) - \tilde{f}_j) \\ &\leq 2T(k) \left[ \sum_{l,j} y_{l,j}^{d_l}(k) q_{l,j}(k) - \sum_{l,f_j} \tilde{\delta}_{l,j} \tilde{f}_j P_{l,j} q_{l,j}(k) \right. \\ &\quad \left. - K \sum_j b_j (f_j(k) - \tilde{f}_j) \right] + 2C \quad (37) \\ &\leq 2T(k) \left[ \sum_{l,j} y_{l,j}^{d_l}(k) q_{l,j}(k) - \sum_{l,j} \tilde{y}_{l,j}^{d_l} q_{l,j}(k) \right. \\ &\quad \left. - K \sum_j b_j (f_j(k) - \tilde{f}_j) \right] + 2C. \end{aligned} \quad (38)$$

Equation (37) follows because  $\{\tilde{f}_j\}$  and  $\{\tilde{\delta}_{l,j}\}$  satisfy (5). Equation (38) follows because  $\{\tilde{y}_{l,j}^{d_l}\}$  satisfies (4). Adding and subtracting  $2K \sum_{l,j} a_{l,j} T(k) (y_{l,j}^{d_l} + y_{l,j}^{d_l}(k))$  to RHS of equation (38), we have

$$\begin{aligned} \Delta V(k) &\leq 2 \sum_{l,j} T(k) y_{l,j}^{d_l}(k) (K a_{l,j} + q_{l,j}(k)) + 2C \\ &\quad - 2 \sum_{l,j} T(k) \tilde{y}_{l,j}^{d_l} (K a_{l,j} + q_{l,j}(k)) \\ &\quad - 2K \sum_{l,j} a_{l,j} T(k) (y_{l,j}^{d_l}(k) - \tilde{y}_{l,j}^{d_l}) \\ &\quad - 2KT(k) \sum_j b_j (f_j(k) - \tilde{f}_j). \end{aligned} \quad (39)$$

Note that up until this point, we have only used the fact that  $(1 + \epsilon)\mathbf{x} \in \Lambda$  (and various substitutions) to arrive at the upper bound (39). Because of algorithm (12), we have

$$\begin{aligned} \sum_{l,j} T(k) y_{l,j}^{d_l}(k) (K a_{l,j} + q_{l,j}(k)) &= \\ \sum_l x_l^{d_l} T(k) \min_j \{K a_{l,j} + q_{l,j}(k)\}. \end{aligned} \quad (40)$$

Since  $\sum_j \tilde{y}_{l,j}^{d_l} = (1 + \epsilon) x_l^{d_l}$ ,

$$\begin{aligned} (1 + \epsilon) \sum_l x_l^{d_l} T(k) \min_j \{K a_{l,j} + q_{l,j}(k)\} \\ \leq \sum_{l,j} T(k) \tilde{y}_{l,j}^{d_l} q_{l,j}(k) + K \sum_{l,j} a_{l,j} T(k) \tilde{y}_{l,j}^{d_l}. \end{aligned} \quad (41)$$

Thus,

$$\begin{aligned} \Delta V(k) &\leq 2C - 2K \sum_{l,j} T(k) a_{l,j} (y_{l,j}^{d_l}(k) - \tilde{y}_{l,j}^{d_l}) \\ &\quad - 2\epsilon \sum_l x_l^{d_l} T(k) \min_j \{q_{l,j}(k) + K a_{l,j}\} \\ &\quad - 2KT(k) \sum_j b_j (f_j(k) - \tilde{f}_j). \end{aligned}$$

Since  $a_{l,j} \leq a_{\max}$ ,  $b_j \leq b_{\max}$ ,  $T_{\min} \leq T_j \leq T_{\max}$ , and  $T(k) y_{l,j}^{d_l}(k)$ ,  $T(k) \tilde{y}_{l,j}^{d_l} \leq \eta_{\max}$ , we have

$$\begin{aligned} q'_{\max} &= 2C + 2K \sum_{l,j} \eta_{\max} + 2K \sum_{j'} b_{\max} T_{\max} \\ &\geq 2C - 2K \sum_{l,j} T(k) a_{l,j} (y_{l,j}^{d_l}(k) - \tilde{y}_{l,j}^{d_l}) \\ &\quad - 2KT(k) \sum_j b_j (f_j(k) - \tilde{f}_j), \end{aligned}$$

$$\Delta V(k) \leq q'_{\max} - 2\epsilon \sum_l x_l^{d_l} T(k) \min_j \{q_{l,j}(k) + K a_{l,j}\}.$$

Let

$$\nu = \min \left\{ x_l^{d_l} \text{ s.t. } x_l^{d_l} > 0 \right\},$$

i.e.,  $\nu$  is the smallest source rate greater than 0 among all sources in the network. If there is  $l$  such that  $q_{l,j}(k) > q_{\max} = q'_{\max}/(2\epsilon T_{\min} \nu)$  for all  $j$ , then equation (24) holds. Thus,

$$q_{l,j}(k) \leq q_{\max} + \eta_{\max}, \quad \forall l, j, k. \quad (42)$$

The control decision in equation (14) combined with the bound (42) prevents  $Q_{l'}(k)$  from being  $> q_{\max} + \eta_{\max}$  for any  $l', k$ . Thus,  $Q_{l'}(k) \leq q_{\max} + \eta_{\max} \forall l', k$ .

Because of equation (42),  $A(k)T(k) \leq \sum_{l,j} \eta_{\max} q_{\max}$ . By simply adding  $2K \sum_j f_j(k) b_j T(k) > 0$  to eq. (34), we have

$$\begin{aligned} \Delta V(k) &\leq \\ &2A(k)T(k) - 2B(k)T(k) + 2C \\ &- 2\kappa \sum_j T(k)(f_j(k) - p_j)w_j(k) + 2K \sum_j f_j(k)b_j T(k) \end{aligned}$$

If there is  $w_j(k)$  such that  $w_j(k) > \frac{\sum_{l,j} \eta_{\max} q_{\max} + C + 2 \sum_j K b_{\max} T_{\max}}{\kappa(1-p_j)T_{\min}}$ , then  $B(k)T(k) + \kappa \sum_j T(k)(f_j(k) - p_j)w_j(k) - K \sum_j f_j(k)b_j T(k) \geq \sum_{l,j} \eta_{\max} q_{\max} + C + K \sum_j b_{\max} T_{\max}$  by algorithm (13), which implies that  $\Delta V(k) < 0$ . Thus, if  $w_j(k) > \frac{\sum_{l,j} \eta_{\max} q_{\max} + C + K \sum_j b_{\max} T_{\max}}{\kappa(1-p_j)T_{\min}}$  for some  $j, k$ , then  $\Delta V(k)$  satisfies equation (24). Therefore,  $w_j(k) \leq \frac{\sum_{l,j} \eta_{\max} q_{\max} + C + K \sum_j b_{\max} T_{\max}}{\kappa(1-p_j)T_{\min}} \forall j, k$ .

## APPENDIX B: PROOF OF THEOREM 2

Let  $C$  be as defined in eq. (33). Then, by eq. (39), we have

$$\begin{aligned} \Delta V(k) &\leq 2C - 2K \sum_{l,j} T(k) a_{l,j} (y_{l,j}^{d_l}(k) - \hat{y}_{l,j}^{d_l}) \\ &\quad - 2KT(k) \sum_j b_j (f_j(k) - \hat{f}_j), \end{aligned}$$

because of eqs. (40) and (41) and since  $\sum_j \hat{y}_{l,j}^{d_l} = x_l^{d_l}$ . This implies that

$$\begin{aligned} &\lim_{k' \rightarrow \infty} \frac{1}{\sum_{k \leq k'} T(k)} \sum_{k \leq k'} \Delta V(k) \\ &= \lim_{k' \rightarrow \infty} \frac{k'}{\sum_{k \leq k'} T(k)} \frac{1}{k'} \sum_{k \leq k'} \Delta V(k) \\ &\leq 2C - 2K \lim_{k' \rightarrow \infty} \frac{\sum_{k \leq k'} \sum_{l,j} T(k) a_{l,j} (y_{l,j}^{d_l}(k) - \hat{y}_{l,j}^{d_l})}{\sum_{k \leq k'} T(k)} \\ &\quad - 2K \lim_{k' \rightarrow \infty} \frac{1}{\sum_{k \leq k'} T(k)} \sum_{k \leq k'} \sum_j b_j f_j(k) T(k) \\ &\quad + 2K \sum_j \hat{f}_j b_j \end{aligned}$$

Since  $\{q_{l,j}(k), Q_{l'}(t), w_j(k)\}$  are bounded by Theorem 1, we have  $\lim_{k' \rightarrow \infty} \frac{1}{k'} \sum_{k \leq k'} \Delta V(k) = 0$ , which implies that

$$\begin{aligned} &\lim_{k' \rightarrow \infty} \frac{\sum_{k \leq k'} (\sum_{l,j} T(k) a_{l,j} y_{l,j}^{d_l}(k) + \sum_j f_j(k) T(k) b_j)}{\sum_{k \leq k'} T(k)} \\ &\leq \sum_{l,j} a_{l,j} \hat{y}_{l,j}^{d_l} + \sum_j \hat{f}_j b_j + C/K. \end{aligned} \quad (43)$$

In addition, we have

$$\begin{aligned} &\sum_{l,j} T(k) a_{l,j} y_{l,j}^{d_l}(k) + \sum_j f_j(k) b_j T(k) \\ &\geq \sum_{l,j} T(k) a_{l,j} \hat{y}_{l,j}^{d_l} + T(k) \sum_j \hat{f}_j b_j \end{aligned} \quad (44)$$

for each  $k$ . Eq. (44) holds because  $\{\hat{f}_j, \hat{y}_{l,j}^{d_l}\}$  is an optimal solution to eq. (9). Eqs. (44) and (43) show that  $\sum_{l,j} a_{l,j} \hat{y}_{l,j}^{d_l} + \sum_j \hat{f}_j b_j$  is  $\leq$  and  $\geq$  to RHS of eq. (20), respectively. ■

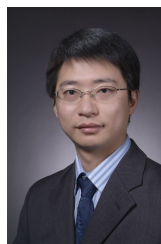
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**Jung Ryu** received his B.S. degree from Northwestern University in Evanston, IL and is now a Ph.D. graduate student in the ECE Department at the University of Texas at Austin. His current research interests include wireless network architectures and design.



**Lei Ying** (M'08) received his B.E. degree from Tsinghua University, Beijing, in 2001, his M.S. and Ph.D in Electrical Engineering from the University of Illinois at Urbana-Champaign in 2003 and 2007, respectively. During Fall 2007, he worked as a Postdoctoral fellow in the University of Texas at Austin. He is currently an Assistant Professor at the Department of Electrical and Computer Engineering at Iowa State University. His research interest is broadly in the area of information networks, including wireless networks, mobile ad hoc networks,

P2P networks, and social networks. He received a Young Investigator Award from the Defense Threat Reduction Agency (DTRA) in 2009, NSF CAREER Award in 2010, and is named Litton Assistant Professor at the Department of Electrical and Computer Engineering at Iowa State University for 2010-2012.



**Sanjay Shakkottai** (SM'11) received his Ph.D. from the ECE Department at the University of Illinois at Urbana-Champaign in 2002. He is with The University of Texas at Austin, where he is currently an Associate Professor in the Department of Electrical and Computer Engineering. He received the NSF CAREER award in 2004. His current research interests include network architectures, algorithms and performance analysis for wireless and sensor networks.