Distributed Power Control and Coding-Modulation Adaptation in Wireless Networks using Annealed Gibbs Sampling

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***Abstract*—In wireless networks, the transmission rate of a link is determined by received signal strength, interference from simultaneous transmissions, and available coding-modulation schemes. Rate allocation is a key problem in wireless network design, but a very challenging problem because: (i) wireless interference is global, i.e., a transmission interferes all other simultaneous transmissions, and (ii) the rate-power relation is non-convex and non-continuous, where the discontinuity is due to limited number of coding-modulation choices in practical systems. In this paper, we consider a realistic Signal-to-Interference-and-Noise-Ratio (SINR) based interference model, and assume continuous power space and finite rate options (coding-modulation choices). We propose a distributed power control and coding-modulation adaptation algorithm using *annealed Gibbs sampling*, which achieves throughput optimality in an arbitrary network topology.**

I. Introduction

Wireless communications have become one of the main means of communications over the last two decades, in the form of both cellular (WWAN) and home/business access point (WLAN) communications [1]. Recently, with the development of data centric mobile devices, e.g., iPhone, we have seen a renewed interest in enabling more flexible wireless networks, e.g., ad hoc networks and peer to peer networks [13]. A key problem in the design of ad hoc wireless networks is link-rate control, i.e., controlling transmission rates of the links. In wireless networks, the transmission rate of a link is determined by received signal strength, interference from simultaneous transmissions, and available coding-modulation schemes. Because wireless interference is global, and the rate-power relation is non-convex [2] and non-continuous, distributed link-rate control in ad hoc wireless networks is a very challenging problem.

One approach in the literature is to assume the link rate is a continuous function of the SINR of the link [8], [11]. For example, a model that has been extensively adopted is to assume $r_{ab} = \frac{1}{2}\log_2(1 + SINR_{ab})$, where r_{ab} is the transmission rate of link (ab). In other words, it assumes that for each SINR level, the capacity achieving coding-modulation is available. Under this assumption, the rate control problem again is formulated as a power control problem where the objective is to find a set of powers to maximize system utility defined upon achievable rates $\sum_{ab} U_{ab}(r_{ab})$, where $U_{ab}(\cdot)$ is the utility function associated with link ab. This problem is also well understood in cellular networks given the recent advances in optimal power control and rate assignment[4], [5], where distributed iterative algorithms are shown to converge to the utility maximizing power allocations, after introducing a small signaling overhead to the cellular air interface. However, these approaches ignore the non-convex nature of the problem and the algorithms proposed here converge to the utility maximizing operating point on *Pareto boundary* of the rate region, assuming all devices have to transmit all the time. In the context of ad hoc networks, such approaches can be highly sub-optimal since the time-sharing, or inter-link scheduling, nature of the problem has to be considered due to highly non-convex nature of the rate-power function. Towards this end, queue-length distributed scheduling is shown to be throughput optimal [6], [7], [10], [9], through the use of MCMC (Markov Chain Monte Carlo) models. These results however assume collision-based interference model, which in general is over-conservative, and assume fixed transmit power and coding-modulation scheme. Both transmit power and coding-modulation can be adaptively chosen in practical systems. For example, in 802.11g, eight rate options are available, and many 802.11 chip solutions have capability of packet to packet power control with very good granularity (0.5dBm).

In this paper, we extend the framework in [6], [7], [10], [9] to develop a distributed joint power control and rate scheduling algorithm for wireless networks based on the SINR-based interference model. We assume that each node has a finite number of coding-modulation choices, but can continuously control transmit power. We propose a distributed algorithm that maximizes the sum of weighted link rates $\sum_{(ab)} w_{ab}r_{ab}$, where r_{ab} is the rate of link ab. The main results of this paper are summarized below:

- We consider realistic SINR-based interference model, where a transmission interferes with all other simultaneous transmissions in the network. Our algorithm decomposes network-wide interference to local interference by properly choosing a "neighborhood" for each node, and bounding the interference from non-neighbor nodes.

- We assume continuous power space and finite coding-modulation choices (rate options). The objective of the algorithm is to find a power and coding-modulation configuration that maximizes the sum of weighted link-rates

$$\begin{array}{ll} \max_{\mathbf{p},\mathbf{m}} & \sum_{(ab)\in\mathcal{E}} q_{ab}(t) r_{ab}(m_{ab},\mathbf{p}) \\ \text{subject to} & \sum_{b:(ab)\in\mathcal{E}} p_{ab} \le p_a^{\max}, \forall a \in \mathcal{V}, \end{array} \quad (1)$$

  where $q_{ab}(t)$ is the queue length of link (ab) at time slot t,[1] $\mathbf{p}$ is a vector containing the power levels of all the links in the network, $p^{\max}$ is the maximum power constraint, and m_{ab} is the coding-modulation scheme. Due to the nonconvexity and discontinuity of $r_{ab}(\cdot)$, optimization problem (1) is very hard to solve in general. Motivated by recent breakthrough of using MCMC to solve MaxWeight scheduling in a completely distributed fashion, we propose a power and coding-modulation update algorithm that emulates a Gibbs sampler over a Markov chain with a continuous state space (the power level of a transmitter is assumed to be continuously adjustable).
- The algorithm based on the Gibbs sampling may be trapped in a local-optimal configuration for an extended period of time. To overcome this problem, we exploit the technique of simulated annealing to speed up the convergence to the optimal power and coding-modulation configuration. The convergence of the algorithm under annealed Gibbs sampling is proved. From the best of our knowledge, this is the first algorithm that uses annealed Gibbs sampling in a distributed fashion with continuous sample space and has provable convergence.

II. System Model

We consider a wireless network with single-hop traffic flows. The network is modeled as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes, and $\mathcal{E}$ is the set of directed links. Let $n = |\mathcal{E}|$ denote the number of links. We assume that time is slotted. Each transmitter a maintains a buffer for each outgoing link (ab), if there is a flow over link (ab). Note that even there are multiple flows over link (ab), a single queue is sufficient for maintaining the stability of the network. The queue length in time slot t is denoted by $q_{ab}(t)$. Each transmitter a has limited total transmit power $p_a^{\max}$, and $p_{ab}(t)$ denotes the transmit power of link (ab) at time slot t.

We assume all links have stationary channels, and each transmitter a can tune its transmit power continuously from 0 to $p_a^{\max}$, but the number of feasible coding-modulation choices is finite. Each coding-modulation associates with a fixed data rate, and a minimum SINR requirement. Thus, the data rate of a link is a step function of the SINR of the link. The SINR of link (ab) is

$$\gamma_{ab}(t) = \frac{p_{ab}(t) g_{ab}}{n_b + \sum_{(xy)\in\mathcal{E},(xy)\neq(ab)} p_{xy}(t) g_{xb}}, \quad (2)$$

where n_b is the variance of Gaussian background noise experienced by node b, and g_{ab} is the channel gain from node a to node b. In this paper, all n_bs and g_{ab}s are assumed to be fixed, i.e., we consider stationary channels.

Denote by $r_{ab}(t)$ the transmission rate of (ab) at time slot t, and $A_{ab}(t)$ the number of bits that arrive at the buffer of the transmitter of link (ab) at the end of time slot t. Then, the queue length $q_{ab}(t)$ evolves as following:

$$q_{ab}(t+1) = [q_{ab}(t) - r_{ab}(t)]^+ + A_{ab}(t), \quad (3)$$

where $[x]^+ = \max\{0, x\}$.

Let $\mathcal{P} \subset \mathbb{R}^n$ denote the set of all feasible power configurations of the network, i.e.,

$$\mathcal{P} = \{\mathbf{p} : \sum_{b:(ab)\in\mathcal{E}} p_{ab} \le p_a^{\max}, p_{ab} \ge 0\}.$$

For each link (ab), given a power configuration $\mathbf{p}$, the SINR of the link γ_{ab} is determined by equality (2). The transmission rate can be written as $r_{ab}(m_{ab}, \mathbf{p})$, where m_{ab} is the coding-modulation scheme.

In this paper, we assume a transmitter always selects the coding-modulation scheme with the highest rate under the given SINR. So m_{ab} is a function of $\mathbf{p}$, and rate r_{ab} can be written as a function of $\mathbf{p} : r_{ab} = r_{ab}(\mathbf{p})$. Then, we define $\mathcal{R}$ as the set of achievable rate vectors under feasible power configurations and modulations, i.e.,

$$\mathcal{R} = \{\mathbf{r}(\mathbf{p}) : \mathbf{p} \in \mathcal{P}\}.$$

The *capacity region* of the network is the set of all arrival rate vectors $\boldsymbol{\lambda}$ for which there exists a power control algorithm that can stabilize the network, i.e., keep the queue lengths from growing unboundedly. It is well known that the capacity region is [12]:

$$\Lambda = \{\boldsymbol{\lambda} | \exists \boldsymbol{\mu} \in Co(\mathcal{R}), \boldsymbol{\lambda} \prec \boldsymbol{\mu}\}, \quad (4)$$

where $Co(\mathcal{R})$ is the convex hull of the set of achievable rates with feasible power configurations, and $\prec$ denotes componentwise inequality. A power control and coding-modulation adaptation algorithm is said to be *throughput optimal* if it can stabilize the network for all arrival rates in the capacity region Λ.

It is well-known that if a rate control algorithm can solve the MaxWeight problem [12] for each time slot, then the algorithm is throughput optimal. The focus of this paper is to develop a power-control and coding-modulation adaptation algorithm to solve the following MaxWeight problem:

$$\begin{array}{ll} \max & \sum_{(ab)\in\mathcal{E}} r_{ab}(\mathbf{p}) q_{ab}(t) - \epsilon \sum_{(ab)\in\mathcal{E}} p_{ab} \\ \text{subject to} & \mathbf{p} \in \mathcal{P}. \end{array} \quad (5)$$

Recall that since $r_{ab}(\mathbf{p})$ is a step function of p_{ab}, multiple power configurations may result in the maximum weighted

[1] We use $q_{ab}(t)$ as the link weight so that the algorithm is throughput optimal when problem is solved at each time slot.

sum. We therefore added a penalty function $-\epsilon \sum_{(ab)} p_{ab}$ with a small ϵ in the objective function to minimize the aggregated power when the weighted sum is maximized.

III. Algorithm

A. Neighborhood and Virtual Rate

To overcome the global interference, we note that because of channel attenuation, interference caused by a remote transmitter in general is negligible. We therefore define a neighborhood for each node a. We say a node b is a *one-hop neighbor* of node a if $\max\{g_{ab}, g_{ba}\} \geq \alpha$, i.e., the channel gain is above certain threshold. Denote by $\mathcal{N}^1(a)$ the set of one-hop neighbors of node a, where the superscript indicates it is the set of one-hop neighbors. We further denote by $\mathcal{N}^2(a)$ the set of two-hop neighbors of node a, i.e., node b belongs to $\mathcal{N}^2(a)$ if $b \notin \mathcal{N}^1(a)$, and $b \neq a$, and there exists a node c such that $c \in \mathcal{N}^1(a)$ and $b \in \mathcal{N}^1(c)$.

Then, we define $\Upsilon_{ab}(t)$ to be

$$\Upsilon_{ab}(t) = \sum_{(xy):x \in \mathcal{N}^1(b)} p_{xy}(t) g_{xb} + \hat{n}_b$$

which is called *noise+partial-interference* at node b for link (ab). Notice that $\hat{n}_b = n_b + \xi_{ab}$, where ξ_{ab} is an upper bound on the interference experienced at node b from the non-neighboring transmitters of b. We assume ξ_{ab} is known. By including this upper bound ξ_{ab} in the SINR computation, we guarantee that the SINR of link (ab) is a function of its neighbors' transmit powers and is independent of non-neighbor nodes. This localizes the interference.

Given the definition of noise+partial-interference, the virtual rate of link (ab) is defined to be

$$\tilde{r}_{ab}(t) = r_{ab}\left(m_{ab}, \frac{p_{ab} g_{ab}}{\Upsilon_{ab}(t)}\right). \tag{6}$$

We will later discuss how to determine transmit power based on actual interference.

Observe that although the power level is continuous, and the SINR of neighboring links are continuous, the virtual rate choices are discrete and finite. Suppose (ab) is changing its power and link $(xy), y \in \mathcal{N}^1(a)$, is affected. For each coding-modulation scheme of link (xy), $m \in \mathcal{M}_{xy}$, let $\gamma_{xy,m}$ denote the SINR requirement of m, and $\Upsilon_{xy,m}$ denote the corresponding partial-noise-plus-interference requirement. Assuming the transmit powers of all other nodes are fixed, the power level $p_{ab} \in [0, p_a^{\max}]$, such that $\Upsilon_{xy}(p_{ab}) = \Upsilon_{xy,m}$, is called a *critical power*, which is highest power node a can use for coding-modulation choice m to be feasible over link (xy).

B. Decision Set

To overcome the issue of predefined update sequence in classic Gibbs sampling, we adopt the technique proposed in [9] to generate a *decision set* at the beginning of each time slot.

Definition 1. A decision set $\mathcal{D}$ is a set of transmitters such that, for any two transmitters a and x in $\mathcal{D}$, $x \notin \mathcal{N}^1(a) \cup \mathcal{N}^2(a)$.

Clearly, two transmitters a and x in the decision set are not one-hop or two-hop neighbors. In the proposed algorithm, only the links in the decision set are allowed to update their transmit powers.

C. Required Information

We further assume that node a has the following knowledge:

- the channel gain from node a to its one-hop neighbors, i.e., g_{ay} for all $y \in \mathcal{N}^1(a)$.
- for each link whose receiver is a's one-hop neighbor, i.e., $(xy) : y \in \mathcal{N}^1(a)$:
  - the virtual transmit power of (xy), i.e., $\tilde{p}_{xy}(t-1)$.[2]
  - the channel gain of (xy), i.e., g_{xy}.
  - the virtual partial-interference-plus-noise of (xy), i.e., $\tilde{\Upsilon}_{xy}(t)$.[3]
  - the queue length of (xy), i.e., $q_{xy}(t)$.
  - the feasible modulations of (xy), and the minimum SINR requirement, $\Upsilon_{xy,m}$, of each modulation m.

Notice that $\tilde{p}_{xy}(t-1), q_{xy}(t)$, and $\tilde{\Upsilon}_{xy}(t)$ change over time. We will explain the way node a obtains these values from node x in the algorithm. Further we assume all channel gains are known to a, and are fixed. The feasible modulations and the minimum SINR requirement for each modulation are assumed to be known a-priori, and do not need to be exchanged.

D. Distributed Power Control and Coding Modulation Adaptation Algorithm

We now present the proposed algorithm, where the evolution of power configuration emulates a Gibbs sampler. A simple example showing how the algorithm works is presented in the technical report [14]. To improve the convergence of the Gibbs sampler, we exploit the technique of simulated annealing [3].

We group every T time slots into a super time slot. In each super time slot, we run the algorithm for T times in the background of each node. In t^{th} time slot of a super time slot, the value K is set to be $K_t = \frac{K_0}{\log(2+t)}$, where K_t is the "temperature" in the terminology of simulated annealing, and K_0 is a positive constant that can be tuned to control the convergence of the proposed algorithm.

All nodes maintain virtual power $\tilde{p}$, and the initial power configuration $\tilde{p}_{ab}(0) = 0$, known by all the nodes in the network. At t^{th} time slot of a super time slot, the algorithm works as follows:

(1) **Generating decision set:** Each time slot consists of W control slots at the beginning. A decision set is determined at the end of the W control slots. Only the transmitters in the decision set update their virtual power levels at this time slot. At time slot t, transmitter a contends for being in the decision set as follows:

[2] the real transmit power is determined by the virtual transmit power. The way the virtual power is computed will be described in detail in the algorithm.

[3] calculated based on virtual transmit powers

(i) Node a uniformly and randomly selects an integer backoff time T_a from $[0, W-1]$ and wait for T_a control slots.

(ii) If a receives an INTENT message from another transmitter x such that

$$x \in \mathcal{N}^1(a) \cup \mathcal{N}^2(a)$$

before control slot $T_a + 1$, node a will not be included in the decision set in this time slot. Here, we assume the INTENT message from x has the id of x and the signal is strong enough, so that x's one-hop and two-hop neighbors, e.g., node a, know this INTENT message is sent by x.

(iii) If node a senses a collision of INTENT messages from nodes x, $x \in \mathcal{N}^1(a) \cup \mathcal{N}^2(a)$, a will not be in the decision set in this time slot.

(iv) If node a does not receive any INTENT message from its one-hop or two-hop neighbors before control slot T_a+1, node a will broadcast an INTENT message to its one-hop and two-hop neighbors in control slot $T_a + 1$.

  a) If the INTENT message from node a collides with another INTENT message sent by node $x \in \mathcal{N}^1(a) \cup \mathcal{N}^2(a)$, a will not be in the decision set in this time slot.

  b) If there is no collision, node a will be included in the decision set in this time slot.

We note that W is selected to be large enough so that the collision of the INTENT messages happens with low probability.

(2) **Link selection:** Let d_a denote the outgoing degree of node a, i.e., $d_a = |\{b : (ab) \in \mathcal{E}\}|$. In this step, each transmitter $a \in \mathcal{D}$ selects an outgoing link (ab) to update its virtual power $\tilde{p}_{ab}$ as following:

(i) If there was an active outgoing link (ab) such that $\tilde{p}_{ab}(t-1) > 0$, a will update the power of link (ab) in time slot t, with probability $\frac{1}{d_a}$.

(ii) If there was no active outgoing link (ab) such that $\tilde{p}_{ab}(t-1) > 0$, a uniformly randomly selects a link (ab) from its d_a outgoing links, and then updates its virtual power $\tilde{p}_{ab}$.

(3) **Critical power level computation:** Node a computes the critical power level $\tilde{p}_{c,m}(ab, xy)$ as follows:

(i) Node a computes the critical partial-noise-plus-interference of link (xy) corresponding to each $\gamma_{xy,m}, m \in \mathcal{M}_{xy}$:

$$\tilde{\Upsilon}_{xy,m} = \frac{\tilde{p}_{xy}(t-1)g_{xy}}{\gamma_{xy,m}}.$$

(ii) Node a computes the critical power $\tilde{p}_{c,m}(ab, xy)$, such that when link (ab) uses this power level, the resulting partial-noise-plus-interference of link (xy) is $\tilde{\Upsilon}_{xy,m}$:

$$\begin{aligned}&\tilde{p}_{c,m}(ab, xy)\\&= \min\left\{p^{\max}, \left[\tilde{p}_{ab}(t-1) + \frac{\tilde{\Upsilon}_{xy,m} - \tilde{\Upsilon}_{xy}(t-1)}{g_{ay}}\right]^+\right\}.\end{aligned}$$

(Some modulations of link (xy) need very high SINR, which cannot be achieved even link (ab) reduces its power to 0. For these modulations, we just let $\tilde{p}_{c,m}(ab, xy)$ be zero, we will consider the 0 critical power separately in the following step.)

(4) **Virtual rates computation:** Now for each node a in the decision set $\mathcal{D}$, it computes the virtual rate of link (ab), and the virtual rate of the links whose receiver is a's neighbor as following:

(i) Arrange the critical power levels

$$\{\tilde{p}_{c,m}(ab, xy), \forall (xy) \quad s.t. \quad y \in \mathcal{N}^1(a)\}$$

in ascending order, denoted by

$$0 = \tilde{p}_{c,0} < \tilde{p}_{c,1} < \cdots < \cdots = p_a^{\max}$$

(ii) Compute the SINR of link (xy), when the power of link (ab) is zero:

$$\gamma_{xy}^0 = \frac{\tilde{p}_{xy}(t-1)g_{xy}}{\tilde{\Upsilon}_{xy}(t-1) - \tilde{p}_{ab}(t-1)g_{ay}}.$$

Further, find the coding-modulation of link (xy) with the largest transmission rate corresponding to this SINR. Let the coding-modulations of all the neighboring links of link (ab) be denoted by a vector $\mathbf{m}^0$.

(iii) Given this initial coding-modulation vector $\mathbf{m}^0$, a obtains the coding-modulation vector $\mathbf{m}^i$, corresponding to each critical power $\tilde{p}_{c,i}, i = 1, 2, \cdots$.

(iv) Obtain the rates $\tilde{r}_{(xy)}(\tilde{p}_{c,i})$ related to the coding-modulations $m_{(xy)}^i$ of neighboring links, when

$$\tilde{p}_{ab} \in [\tilde{p}_{c,i}, \tilde{p}_{c,i+1}),\ i = 0, 1, \cdots .$$

Note that for each link, the $m_{(xy)}^i$ is the coding-modulation scheme with the highest transmission rate assuming node a transmits with power $\tilde{p}_{c,i}$.

(v) Compute the virtual local weight, under each critical power level $\tilde{p}_{c,i}$:

$$\tilde{V}_{ab}(\tilde{p}_{c,i}) = \sum_{y \in \mathcal{N}^1(i), (xy) \in \mathcal{E}} \tilde{r}_{xy}(\tilde{p}_{c,i}) q_{xy}$$

for $i = 0, 1, \cdots$, where q_{xy} is the queue length at the beginning of the super time slot.

(5) **Power-level selection:** Let Z_{ab} be the normalization factor defined as $Z_{ab} = \sum_i (e^{-\frac{\epsilon \tilde{p}_{c,i}}{K_t}} - e^{-\frac{\epsilon \tilde{p}_{c,i+1}}{K_t}}) e^{\frac{\tilde{V}_{ab}(\tilde{p}_{c,i})}{K_t}}$. Node a first selects a power interval $[\tilde{p}_{c,i}, \tilde{p}_{c,i+1})$ with following probability:

$$\Pr(\tilde{p}_{ab} \in [\tilde{p}_{c,i}, \tilde{p}_{c,i+1})) = \frac{1}{Z_{ab}} \left(e^{-\frac{\epsilon \tilde{p}_{c,i}}{K_t}} - e^{-\frac{\epsilon \tilde{p}_{c,i+1}}{K_t}}\right) e^{\frac{\tilde{V}_{ab}(\tilde{p}_{c,i})}{K_t}}. \tag{7}$$

Suppose the interval $[\tilde{p}_{c,k}, \tilde{p}_{c,k+1})$ is selected, then node a randomly selects a virtual power level $\tilde{p}_{ab}(t)$ according to the following probability density function (pdf):

$$f_{ab}(p|p \in [\tilde{p}_{c,k}, \tilde{p}_{c,k+1})) = \frac{\epsilon}{K_t} e^{-\frac{\epsilon p}{K_t}} \left(e^{-\frac{\epsilon \tilde{p}_{c,k}}{K_t}} - e^{-\frac{\epsilon \tilde{p}_{c,k+1}}{K_t}}\right)^{-1}, \tag{8}$$

which can be done by using the inverse transform sampling method.

(6) **Information exchange:** If the virtual power of a node has changed, i.e., $\tilde{p}_{ab}(t-1) \neq \tilde{p}_{ab}(t)$, then node a broadcasts $\tilde{p}_{ab}(t)$ to all its one-hop neighbors. Each neighbor y computes the virtual partial-interference-plus-noise of (xy), i.e., $\tilde{\Upsilon}_{xy}(t)$. If $\tilde{\Upsilon}_{xy}(t-1) \neq \tilde{\Upsilon}_{xy}(t)$, node y broadcasts $\tilde{\Upsilon}_{xy}(t)$ to all its one-hop neighbors.

(7) **Update real transmit power:** At the end of a super time slot, node a updates its transmit power for link (ab) to be $p_{ab} = \tilde{p}_{ab}(T)$. Node b then measures the actual SINR γ_{ab} and reports to node a. Node a selects the coding-modulation scheme m_{ab} with the highest rate among those m such that $\gamma_{(ab),m} \leq \gamma_{ab}$. Packets of flow (ab) are transmitted with power p_{ab} and coding-modulation scheme m_{ab}. Note that the real transmit powers are updated only once every T time slots.

E. Analysis

In the classical Gibbs sampler, the state of each link is updated in a sequential manner. In contrast, the Gibbs sampler used in our algorithm is parallelized and distributed, which leverages the distributed characteristic of wireless networks.

The following theorem states that by decreasing the temperature in a controlled manner, the distribution of the sampled power configuration converges to the one putting almost all the density in the optimal power configuration.

Theorem 1. *Let* $U(\tilde{\mathbf{p}}) = \tilde{V}(\tilde{\mathbf{p}}) - \epsilon \sum_{(ab)\in\mathcal{E}} \tilde{p}_{ab}$, *and*

$$U^* = \max_{\tilde{\mathbf{p}}\in\mathcal{P}} U(\tilde{\mathbf{p}}), U_* = \min_{\tilde{\mathbf{p}}\in\mathcal{P}} U(\tilde{\mathbf{p}}), \Delta = U^* - U_*$$

Let $K_0 = 2n\Delta$. *Assume* q_{ab} *are fixed and* $\mathcal{P}^*_\delta$ *is the set of power configurations such that for any* $\tilde{\mathbf{p}} \in \mathcal{P}^*_\delta$,

$$\begin{aligned}&\textstyle\sum_{(ab)\in\mathcal{E}} q_{ab}\tilde{r}_{ab}(\tilde{\mathbf{p}}) - \epsilon \sum_{(ab)\in\mathcal{E}} \tilde{p}_{ab} \\ &\geq (1-\delta)\max_{\tilde{\mathbf{p}}} \textstyle\sum_{(ab)\in\mathcal{E}} q_{ab}\tilde{r}_{ab}(\tilde{\mathbf{p}}) - \epsilon \sum_{(ab)\in\mathcal{E}} \tilde{p}_{ab}.\end{aligned}$$

Then given any $\delta > 0$, $\varepsilon > 0$, *and starting from any initial power configuration,* $\tilde{\mathbf{p}}_0 \in \mathcal{P}$, *there is an* $N \in \mathbb{N}$ *such that if*

$$K_t = \begin{cases} \frac{K_0}{\log(2+t)} & 0 < t < N, \\ \frac{K_0}{\log(2+N)} & N \leq t \end{cases} \tag{9}$$

we have

$$\lim_{t\to\infty} \int_{\tilde{\mathbf{P}}^*_\delta} P(t, \tilde{\mathbf{p}}|0, \mathbf{p}_0) d\tilde{\mathbf{p}} \geq 1 - \varepsilon.$$

Proof: The complete proof can be found in the technical report [14]. ■

Remark 1: The theorem requires that the queue lengths are fixed during the annealing, which is the reason the algorithm uses the queue lengths at the beginning of a super time slot for the entire super time slot.

Remark 2: In the algorithm, we replace the interference from non-neighbor nodes with upper bound ξ. Therefore, when node a changes its transmit power to node b to $\tilde{p}_{ab}(T)$ at the end of a super time slot, the actual rate r_{ab} is at least $\tilde{r}_{ab}(T)$, because the actual interference is smaller than that in the virtual rate computation. Further when the neighborhood is chosen to be large enough, i.e., ξ is small, the optimal configuration based on virtual rate is close to the optimal configuration with global interference. But a large neighborhood increases both the computation and communication complexities.

IV. Conclusion

In this paper, we developed a distributed power control and coding-modulation adaptation algorithm using annealed Gibbs sampling, which achieves throughput optimality in an arbitrary network topology. The power update policy emulates a Gibbs sampler over a Markov chain with a continuous state space. Simulated annealing is exploited in the algorithm to speed up the convergence of the algorithm to the optimal power and coding-modulation configuration. Simulation results (presented in the technical report [14]) demonstrated the superior performance of the proposed algorithm.

Acknowledgement: This work was partially supported by the National Science Foundation under Grant CNS-09-53165, and by the Defense Threat Reduction Agency (DTRA) under Grant HDTRA1-09-1-0055.

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