GLOBAL STABILITY OF INTERNET CONGESTION CONTROL WITH ONE-BIT MARKING

L. Ying, R. Srikant and G. E. Dullerud

Abstract: This paper focuses on the problem of designing globally stable, scalable congestion control algorithms for the Internet. A sufficient condition for global stability has been obtain in (Ying *et al.*, 2004) in term of the increase/decrease parameters of the congestion control algorithm and the price functions. In this paper, the result will be extended to one-bit marking scheme.

Keywords: Network, Global Stability, Time Delay

1. INTRODUCTION

Congestion control in the Internet was introduced in the later 1980s by Van Jacobson (Jacobson, 1988). Jacobson's algorithm which is implemented in the transport layer protocol called TCP (Transmission Control Protocol) has served the Internet well during a time of unprecedented growth. However, the algorithm was designed during a time when the Internet was a relatively small network compared to its size today. Therefore, there has been much interest in reexamining the role of congestion control in the Internet with the goal of enhancing TCP to make it scalable to large networks.

This re-design of congestion control algorithm has received much attention since the work of of Kelly et al (Kelly *et al.*, 1998), where congestion control is viewed as a distributed control algorithm for achieving fair resource allocation in a network with many users. However, such model does not provide insight into how congestion control parameters should be chosen in the presence of feedback delays. A series of papers (Johari and Tan, 2001), (Misra *et al.*, 2000; **?**) (Vinnicombe, 2001), (Vinnicombe, 2002), (Paganini *et al.*, 2001), (Kunniyur and Srikant, 2003) provided such design guidelines by considering the local stability for a general topology network without any restrictions on the network parameters. However, the region of attraction for these local stability results is not yet to be established. Using Razumikhin's theorem, a global, asymptotical stability result has been obtained for a general network with heterogeneous delays in the paper (Ying *et al.*, 2004). A similar result for a slightly different class of single-phase controllers has been obtained in (La *et al.*, 2004) also. One assumption of paper (Ying *et al.*, 2004) is that the price of the route is simply the sum of the prices of the links along that route. In this paper, a similar result is obtained for one-bit marking scheme, where the price of a route is the probability with which a packet will be marked on that route. If the marking probability of link j is p_j, then the price of route l is $1 - \Pi_{j \in l}(1 - p_j)$.

The rest of this paper is organized in follows. In Section 2, a simple model for Jacobson's algorithm that was introduce in (Srikant, 2004) is presented and its local stability in the presence of feedback delays is analyzed. The results in (Srikant, 2004) and (Massoulie, 2002) are summarized and show that TCP-Reno is not stable when the capacity per user is large or if the feedback delay is large. In Section 3, the global stability condition for a general topology network with heterogeneous delays obtained in the paper (Ying *et al.*, 2004) is introduced. One assumption of this result is that the price of a route is the sum of the price of the links of that route. In Section 4, the result is extended to one-bit marking scheme and a sufficient condition for global stability condition is obtained. In Section 5, the performance of a network with random disturbance is studied with simulations. In Section 6, the concluding remarks is provided.

2. A SIMPLE MODEL FOR TCP-RENO

Congestion control is implemented in the Internet using a window flow control algorithm. A source's window is the maximum number of unacknowledged packets that the source can transmit at any time. Let $W_i(t)$ denote the window size of source i, then the idea of the TCP-Reno algorithm is as follows:

- Upon receipt of an ACK, the source i increases its current window size as follows:
$$W_i(t) \leftarrow W_i(t) + \frac{1}{W_i(t)}.$$
- Upon being informed of a loss, the source decreases its window size by a factor of two:
$$W_i(t) \leftarrow \frac{W_i(t)}{2}.$$

Now, consider N TCP-Reno sources, all with the same RTT, accessing a single link. Let T_i denote RTT (Round Trip Time) of source i, let $q_i(t)$ be the fraction of packets lost at time t. Then, the congestion avoidance phase of TCP-Reno can be modelled as following delay differential equation:

$$\dot{W}_i = \frac{x_i(t-T)(1-q(t))}{W_i} - \beta x_i(t-T)q(t)W_i(t)$$

where $x_i(t) = W_i(t)/T$ is the transmission rate and the parameter β is the decrease factor. Substituting for $W_i(t)$ in terms of $x_i(t)$ gives

$$\dot{x}_i(t) = \frac{x_i(t-T)(1-q(t))}{T^2 x_i(t)} - \beta x_i(t-T)q(t)x_i(t) \quad (1)$$

where $q(t)$ is the loss probability and a function of the arrival rate at the link. Thus, let

$$q(t) = f(y(t-T)),$$

where $f(\cdot)$ is an increasing function and $y(t)$ is the total arrival rate at the link and is given by

$$y(t) = \sum_{j=1}^{N} x_j(t).$$

To simplify the analysis, assume that $f(y)$ is of the form suggested in (Kunniyur and Srikant, 2003):

$$f(y) = \left(\frac{y-c}{y}\right)^{+}.$$

Thus, this form of $f(y)$ can be interpreted as a fluid approximation to the loss probability: when the arrival rate is less than the link capacity, then the number of dropped packets is zero. On the other hand, when the arrival rate is greater than or equal to the link capacity, then the rate at which packets are dropped is equal to the arrival rate minus the service rate. For TCP congestion control to work, packet loss must necessarily occur. Therefore, we make the assumption that the queue is nearly full most of the time. Thus, we can approximate the queueing delay as B/C, where B is buffer size at the router. So for all users, the RTT is a constant

$$T = T_p + B/C,$$

where T_p is the propagation delay. Deriving the global asymptotical stability condition for (1) is hard. Therefore, the system is linearized around its equilibrium point. Using $\hat{x}_i$ and $\hat{q}_i$ to denote the equilibrium values, where

$$\hat{x}_i = \sqrt{\frac{1-\hat{q}}{\beta\hat{q}}}\frac{1}{T}, \quad (2)$$

and defining $z_i(t) = x_i(t) - \hat{x}_i$. Thus, the linearized form of the congestion control algorithm is given by

$$\dot{z}_i(t) + a z_i(t) + b z_i(t-T) = 0,$$

where $a = 2\beta\hat{q}\hat{x}_i$ and $b = \beta\hat{x}_i$.

From Hayes' lemma (Hale and Lunel, 1991), the linearized delay-differential equation describing TCP-Reno's dynamics is stable if one of the following conditions is satisfied:

- $a \geq b$
- $a < b$ and
$$bT\sqrt{1-\frac{b^2}{a^2}} < \arccos\left(-\frac{a}{b}\right).$$

For the first condition to be satisfied, it requires $\hat{q} \geq 1/2$. This is not a practical scenario since it requires at least half of the packets to be dropped at the router. The second condition can be written as

$$\frac{c}{N}T < \frac{1}{\beta}\frac{(1-\hat{q})\arccos(-2\hat{q}_i)}{\sqrt{1-4\hat{q}^2}}.$$

Note the equilibrium relationship (2) can be rewritten as

$$\frac{(1-\hat{q})^3)}{\hat{q}} = \left(\frac{c}{N}T\right)^2.$$

If let c/N (which is simply the capacity per user) be a constant and increase the RTT, then the right-hand side of the stability condition can be approximated by letting $\hat{q} = 0$ which gives the following condition for stability

$$\frac{c}{N}T < \frac{\pi}{2\beta}.$$

Clearly, this condition will be violated as T increases or c/N increases. From the above analysis, we can conclude that TCP-Reno is not a scalable protocol, i.e., its stability is compromised if either the RTT of the users is large or if the available capacity per user at the router is large.

3. GLOBAL STABILITY RESULT FOR ADDITIVE PRICE FUNCTION

From above section, we know that TCP-Reno is not stable when the RTT or the available capacity per user is large. So it is interesting to design a global stable and scalable congestion control algorithm. A new technique using Razumikhin's theorem has been developed in paper (Ying *et al.*, 2004) which gives a global stability condition for a general topology network with heterogeneous delays in term of the control parameters. We first introduce some notation to describe the network with delays, which will be used in this section and next one. Let $d_f(i,l)$ be the forward delay from source i to link l, and $d_b(i,l)$ be the backward delay from link l to source i. Denote $T_i = d_f(i,l) + d_b(i,l)$ the round trip time (RTT) for source i. We assume that the RTT is a constant and not time-varying. This is a reasonable assumption if the price functions $f_l(y_l)$ are designed such that the users get early feedback about congestion so they react before queue buildup occurs at the routers. In that case, the RTT is simply equal to the propagation delay which is constant.

We consider the following TCP-like congestion control algorithm suggested in (Vinnicombe, 2002):

$$\dot{x}_i(t) = \kappa_i x_i\left(t - T_i\right)\left(\frac{a}{x_i^n(t)} - b q_i(t) x_i^m(t)\right), \quad (3)$$

where

$$q_i(t) = \sum_{l \in i} p_l(t - d_r(i,l)),$$

$$p_l(t) = f_l(y_l)),$$

$$f_l(y) = \left(\frac{y}{c_l}\right)^h \quad (4)$$

and

$$y_l(t) = \sum_{k: l \in k} x_k(t - d_f(i,l)).$$

Here a, b and h are positive real number, and m, n are real numbers that satisfy $m + n > 0$. In the above set of equations, x_i is the rate at which source i transmits data, y_l is the arrival rate at link l, p_l is price of link l, q_i is the price of source i's route and $f_l(y)$ is the function of the link arrival rate which is used to compute p_l. The price of a route is simply the sum of the prices of the links along its path. Also define the quantity $d := \max_j T_j$ which will be useful later.

It has been shown in (Ying *et al.*, 2004) that if $m + n > h$, then $x(t) > 0$ for all t whenever the initial conditions are non-zero. So we can define the functions:

$$W_j(t) = \frac{1}{2}\left(\log x_j(t) - \log \hat{x}_j\right)^2$$

which is well-defined provided when the network model has a nonzero initial condition. Also define the function

$$W(t) = \max_j W_j(t). \quad (5)$$

The paper shows that when $m+n > h$, there exists an $\alpha > 1$ such that $W(t)$ decreases whenever

$$\alpha^2 W(t) > \max_{t-d \le r \le t} W(r).$$

It then follows from Razumikhin's theorem that the above congestion control algorithm is global asymptotically stable.

Theorem 1. (Ying *et al.*, 2004) If $m + n > h > 0$, then the network model in (3) is globally asymptotically stable.

4. EXTENSION TO PACKET MARKING

In above section, we assumed that the price of a route is the sum of the prices of the links in the route, and this information can be conveyed to the source using some protocol. However, today's Internet protocols allow only one bit of information per packet to convey link prices back to the source. To deal with this situation, it is convenient to assume that the link prices are real numbers in the interval $[0, 1]$. Thus, the link price p_l can be thought of as the probability with which a link marks a packet. Marking refers to the action of the link by which it flips a bit in the packet header from a 0 to 1 to indicate congestion. Instead of the sum of the link price, let q_i denote the probability with which a packet is marked on route i. Thus,

$$q_i(t) = 1 - \Pi_{l: l \in i}(1 - p_l(t - d_b(i,l))). \quad (6)$$

Furthermore, it is assumed that the marking probability of link l is

$$p_l(t) = 1 - \exp\left(-\zeta_l\left(\frac{y_l(t)}{c_l}\right)^h\right). \qquad (7)$$

Without causing confusion, x_i and q_i are used to denote $x_i(t)$ and $q_i(t)$ in the rest of this paper. Now, define the congestion control algorithm with one-bit marking as follows:

$$\dot{x}_i = \kappa_i x_i(t - T_i)\left(\frac{a_i(1 - q_i)}{x_i^n} - b_i x_i^m q_i\right) \qquad (8)$$

Furthermore, define functions

$$g_j(x) = \left(\frac{(1 - \hat{q}_j)^{x^{\alpha h}} \hat{q}_j}{(1 - \hat{q}_j)(1 - (1 - \hat{q}_j)^{x^{\alpha h}})}\right)^{1/(m+n)} \qquad (9)$$

$$G_j(x) = \mathrm{sgn}(x - 1)(g_j(x) - 1/x). \qquad (10)$$

It is easy to show that if $0 < \hat{q}_j < 1$, $g_j(x)$ is a strictly deceasing function of x and $g_j(1) = 1$. Lemma 2 shows that if the initial conditions satisfy some conditions, it will impose upper and lower bounds on the functions $x_j(t)$ for all j and all t.

Lemma 2. If there exist $\gamma > 1$ and $\alpha > 1$ such that $x_j(t)/\hat{x}_j \in [1/\gamma, \gamma]$ for $t \in [-T_j, 0]$, and $G_j(x) > 0$ holds when $x \in [1/\gamma, \gamma]$ except $x = 1$ for all j, then each of the flow x_j satisfies

$$1/\gamma \leq x_j(t)/\hat{x}_j \leq \gamma$$

for all $t \geq -d$.

Proof: Suppose that some flow x_i violates the inequality on the interval $(0, \infty)$, then by continuity of the function x_j and their derivatives, there must exists a time $t > 0$ such that

(a) $1/\gamma \leq x_j(t)/\hat{x}_j \leq \gamma$ for all $t \in [-d, t]$;
(b) At time t, one of following conditions holds

$$x_i(t)/\hat{x}_i = 1/\gamma \text{ and } \dot{x}_i(t) < 0 \qquad (11)$$

or

$$x_i(t)/\hat{x}_i = \gamma \text{ and } \dot{x}_i(t) > 0. \qquad (12)$$

We will show that this leads to a contradiction.

Recall that the arrival rate at any link l is defined as

$$y_l(t) = \sum_{k:l\in k} x_k(t - d_f(i, l)),$$

then using condition (a) above we have for $r \in [-d, t]$ that

$$\left(\frac{\hat{y}_l}{\gamma c_l}\right)^h \leq \left(\frac{y_l(r)}{c_l}\right)^h \leq \left(\frac{\hat{y}_l \gamma}{c_l}\right)^h.$$

From equations (6) and (7), we have that

$$q_i(t) = 1 - \exp\left(-\sum_{l:l\in i}\left(\zeta_l \frac{y_l(t - d_b(i, l))}{c_l}\right)^h\right),$$

thus,

$$1 - (1 - \hat{q}_i)^{\gamma^{-h}} \leq q_i(t) \leq 1 - (1 - \hat{q}_i)^{\gamma^h}. \qquad (13)$$

Now suppose that (12) holds, then $x_i(t) = \hat{x}_i\gamma$ and $\dot{x}_i > 0$, but from the system equation and the above lower bound on $q_i(t)$ we have

$$\dot{x}_i = \kappa_i x_i(t - T_i)\left(\frac{a_i}{x_i^n}(1 - q_i) - b_i x_i^m q_i\right)$$
$$= \frac{a_i\kappa_i x_i(t - T_i)(1 - q_i)}{x_i^n}\left(1 - \frac{b_i x_i^{m+n} q_i}{a_i(1 - q_i)}\right).$$

Then, from lower bound of (13),

$$\begin{aligned}
&1 - \frac{b_i x_i^{m+n} q_i}{a_i(1 - q_i)} \\
&< 1 - \frac{b_i}{a_i}(\gamma\hat{x}_i)^{m+n}\frac{\left(1 - (1 - \hat{q}_i)^{\gamma^{-h}}\right)}{(1 - \hat{q}_i)^{\gamma^{-h}}} \\
&= 1 - \frac{(1 - \hat{q}_i)\gamma^{m+n}}{\hat{q}_i}\frac{\left(1 - (1 - \hat{q}_i)^{\gamma^{-h}}\right)}{(1 - \hat{q}_i)^{\gamma^{-h}}} \\
&= 1 - \left(\frac{1}{\gamma} g_i\left(\left(\frac{1}{\gamma}\right)^{1/\alpha}\right)\right)^{-(m+n)} < 0,
\end{aligned}$$

where the last inequality holds because $g_i(\gamma^{-1/\alpha}) < g_i(\gamma^{-1}) < \gamma$. Thus, $\dot{x}_i(t) < 0$ and (12) cannot hold. Similarly we can show that a contradiction arises when (11) holds. Hence we conclude that no such time t exists and thus the sought bounds must be valid for all flows for all time. ■

Now we suppose that the condition of Lemma 2 holds, then $x_i(t) > 0$ for all t and $W(t)$ defined by equation (5) is well-defined. Same as the additive price case, we will prove there exists $\alpha > 1$ such that $W(t)$ decreases whenever

$$\alpha^2 W(t) > \max_{t-d\leq r\leq t} W(r). \qquad (14)$$

Lemma 3. Under the conditions of Lemma 2, if at time t the Lyapunov function W satisfies the condition (14), and that index i is such that $W(t) = W_i(t)$. Then

$$B_i^{-\beta}\hat{x}_j < x_j(r) < B_i^{\beta}\hat{x}_j, \qquad (15)$$

for all $r \in [t - d, t]$, where $B_i = \frac{x_i(t)}{\hat{x}_i}$ and $\beta = \mathrm{sgn}(\log B_i)\alpha$.

Proof: See Lemma 4 of paper (Ying *et al.*, 2004). ■

Lemma 4. If the conditions of Lemma 3 hold, then $\dot{W}_i(t) < 0$.

Proof: The derivative $\dot{W}_i(t)$ is given by

$$\dot{W}_i(t) = \frac{\dot{x}_i}{x_i}\log\left(\frac{x_i}{\hat{x}_i}\right) = \frac{\dot{x}_i}{x_i}\log B_i,$$

and thus to prove the result we need to show that $\dot{x}_i(t)\log B_i < 0$. From (8) we see that

$$\dot{x}_i = \frac{\kappa_i a_i(1-q_i)x_i(t-T_i)}{x_i^{n_i}}\left(1 - \frac{b_i q_i x^{m_i+n_i}}{a_i(1-q_i)}\right).$$

Since $x_i(t-T_i)$, x_i and q_i are all positive it is sufficient to show that

$$\left(1 - \frac{b_i q_i}{a_i(1-q_i)}x^{m_i+n_i}\right)\log B_i < 0. \qquad (16)$$

We can show this by considering the following two cases. Suppose first that $B_i > 1$, then we have to show that

$$1 - \frac{b_i q_i}{a_i(1-q_i)}x^{m_i+n_i} < 0.$$

Using Lemma 3 and follow the steps of Lemma 2, we can get

$$1-(1-\hat{q}_i)^{\beta^{-\alpha h}} \le q_i \le 1-(1-\hat{q}_i)^{\beta^{\alpha h}}$$

and

$$1 - \frac{b_i q_i}{a_i(1-q_i)}x^{m_i+n_i} = 1 - \left(\frac{1}{B_i}g\left(\frac{1}{B_i}\right)^{-(m+n)}\right) < 0.$$

Similarly, if $B_i < 1$, we have

$$1 - \frac{b_i q_i}{a_i(1-q_i)}x^{m_i+n_i} > 0.$$

■

Theorem 5. Under the condition of Lemma 2, the network model in (8) is globally asymptotically stable.

Proof: With $W(t)$ and $W_j(t)$ as defined above, it is easy to show that, for every $t \ge 0$,

$$\limsup_{a\to 0^+}\frac{1}{a}W(t+a) - W(t) < 0$$

whenever $\alpha^2 W(t) > \max_{t-d\le r\le t} W(r)$. Thus, the result follows from Razumikhin's theorem (Hale and Lunel, 1991). ■

From above theorem, we know that the global stability of a network depends on whether $G_i(x) > 0$ holds for $x \in [\hat{x}_i/\gamma, \hat{x}_i\gamma]$. Note that function $g_i(x)$ is determined by $m+n$, h and $\hat{q}_i$. So if $\hat{q}_i$ and $m+n$ are fixed, then the problem is how to choose h to make the network globally stable. It is hard to get an exact expression for h because $g_i(x)$ is a nonlinear function. What we can do is guessing one first, then numerically verifying it. The following properties of $g_i(x)$ will help the guess:

(1) Given two functions $G_i^1(x)$ and $G_i^2(x)$, and suppose $m^1+n^1 = m^2+n^2$ and $\hat{q}_i^1 = \hat{q}_i^2$. If $h^1 > h^2$, then

$$G_i^1(x) \ge G_i^2(x)$$

for all $x > 0$. It implies that if there exists h^* such that the network is globally stable when $h = h^*$, then the network is globally stable for any $0 < h \le h^*$.

(2) One necessary condition of $G_i(x) > 0$ is

$$\frac{d}{dx}g_i(x)|_{x=1} > \frac{d}{dx}\frac{1}{x}|_{x=1} = -1,$$

which is equivalent to

$$\alpha h < -(m+n)\frac{\hat{q}_i}{\log(1-\hat{q}_i)}.$$

Because α can be chosen as arbitrarily near 1, the above condition can be further simplified to

$$h < -(m+n)\frac{\hat{q}_i}{\log(1-\hat{q}_i)}, \qquad (17)$$

So given $\hat{q}_i$, $m+n$ and γ, we first choose a h according to (17) to see whether $G_i(x) > 0$ holds. If it doesn't, we decrease h until $G_i(x) > 0$ holds.

For example, we consider a TCP-Reno like algorithm, where $m = n = 1$. Suppose $\hat{q}_i = 0.1$, $\alpha = 1.001$ and $\gamma = 4$, from condition (17), $h < 1.8982$. Choose $h = 1.5$, the function $G_i(x)$ is numerically calculated in Fig. 1. So the network

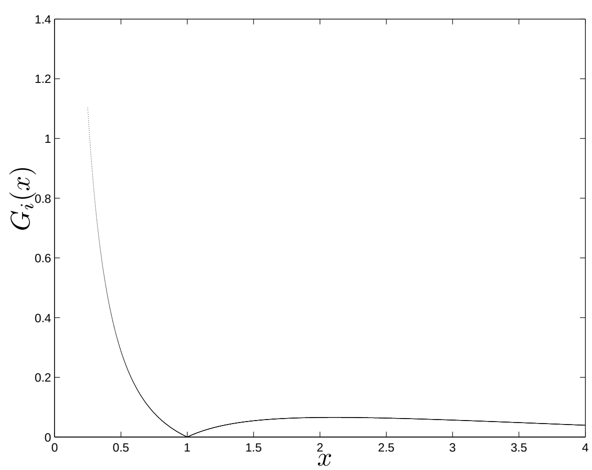

Fig. 1. $G_i(x)$ with $m = n = 1$ and $h = 1.5$.

is globally, asymptotically stable if $h \le 1.5$.

5. SIMULATION

So far in the paper, we have provided a sufficient condition for global stability for the network with one-bit marking scheme. This section is devoted to further investigations of our algorithm, now however using simulation. We consider a single link network with 50 users and use a stochastic discrete-time model:

$$x_i[k+1] = x_i[k] + \kappa_i S(1 - p_i - 0.1 x_i[k] p_i).$$

Here, time step $S = 1/\hat{x}_i$, and in each time slot, user i transmits Poisson$(x_i S)$ packets. One-bit marking is used at the link and each packet will be marked with probability $p = 1 - \exp(-(y/c)^h)$. The users measure the fraction of packets that are marked in a fixed time interval to infer the price p_i. We choose two different hs — $h = 0.8$ and $h = 2$, where the network is globally stable when $h = 0.8$ and locally stable when $h = 2$. Let $\bar{x}(t) = \sum_i x_i(t)/50$, we calculate

$$R = \frac{\text{Standard Deviation of } \bar{x}}{\text{Mean of } \bar{x}}$$

for $\kappa_i \in [0.1, 4]$. The results are as follows:

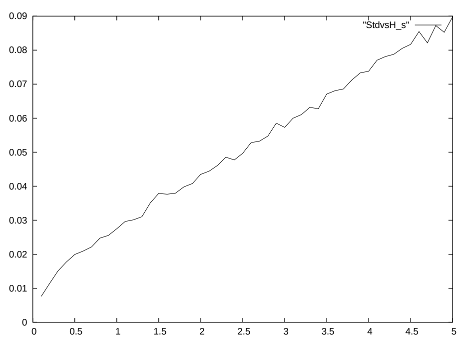

Fig. 2. R vs κ when $h = 0.8$

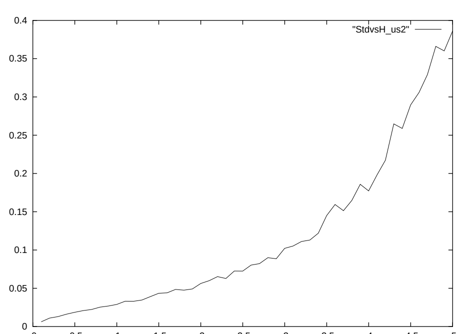

Fig. 3. R vs κ when $h = 2$

Compare Fig. 2 and Fig. 3, R increases very slow when $h = 0.8$ and increases fast when $h = 2$. So from this simulation, we can see that the network is more robust under then random disturbance when it is globally stable.

6. CONCLUSIONS

An important open problem in the study of Internet congestion control has been the design of congestion controllers which are globally stable and scalable. In this paper, we extend our original result in paper (Ying *et al.*, 2004) to one-bit marking scheme. The condition is also in term of the increase/decrease parameters of the congestion control algorithm and the price function used at the link. Also the simulation shows that a globally stable network performs more robust under the random disturbance compared to the network that is only locally stable.

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