On Delay Constrained Multicast Capacity of Large-Scale Mobile Ad-Hoc Networks

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Abstract—This paper studies the delay constrained multicast capacity of large scale mobile ad hoc networks (MANETs). We consider a MANET that consists of \( n_s \) multicast sessions. Each multicast session has one source and \( p \) destinations. Each source sends identical information to the \( p \) destinations in its multicast session, and the information is required to be delivered to all the \( p \) destinations within \( D \) time-slots. Assuming the wireless mobiles move according to a two-dimensional i.i.d. mobility model, we first prove that the capacity per multicast session is

\[
O\left( \frac{1}{\sqrt{n_s \log(n_s,p)}} \right)
\]

\(^1\) We then propose a joint coding/scheduling algorithm achieving a throughput of

\[
\Theta\left( \min\left\{ 1, \sqrt{\frac{D}{n_s p}} \right\} \right).
\]

Our simulation results suggest that the same scaling law also holds under random walk and random waypoint models.

I. INTRODUCTION

Wireless technology has provided an infrastructure-free and fast-deployable method to establish communication, and has inspired many emerging networks such as mobile ad hoc networks (MANETs), which has broad potential applications in personal area networks, emergency/rescue operations, and military battlefield applications. Over the past few years, there has been a lot of interest in understanding the capacity of MANETs under a range of mobility models [1]–[18]. Most of these work assumes unicast traffic flows and studies the unicast capacity. However, multicast flows are expected to be predominant in many of emerging applications. For example, in battlefield networks, commands need to be broadcast in the network or sent to a specific group of soldiers. In a wireless video conference, the video needs to be sent to all participants. To support these emerging applications, it is imperative to have a fundamental understanding of the multicast capacity of wireless networks. In [19], [20], the authors proved that the multicast capacity of a static ad hoc network is

\[
O\left( \frac{1}{\sqrt{n_s \log(n_s,p)}} \right)
\]

per multicast session. In [21], the authors investigated the multicast capacity of delay tolerant networks without delay constraints. In [22], the multicast-capacity and delay tradeoff is established for a specific routing/scheduling algorithm.

\(^1\)Given non-negative functions \( f(n) \) and \( g(n) \): \( f(n) = O(g(n)) \) means there exist positive constants \( c \) and \( m \) such that \( f(n) \leq cg(n) \) for all \( n \geq m \); \( f(n) = \Omega(g(n)) \) means there exist positive constants \( c \) and \( m \) such that \( f(n) \geq cg(n) \) for all \( n \geq m \); \( f(n) = o(g(n)) \) means that both \( f(n) = \Omega(g(n)) \) and \( f(n) = O(g(n)) \) hold; \( f(n) = \omega(g(n)) \) means that \( \lim_{n \to \infty} f(n)/g(n) = 0 \); and \( f(n) = \omega(g(n)) \) means that \( \lim_{n \to \infty} g(n)/f(n) = 0 \).

In this paper, we study the multicast capacity of large-scale MANETs under a general delay constraint \( D \). The multicast problem differs from the unicast problem in the following aspects:

- Each multicast session has multiple destinations, so the probability that a packet is within the transmission range of its destination(s) is higher than that in the unicast scenario. On the other hand, in the multicast scenario, the information needs to be transmitted reliably from a source to all its destinations, which requires more transmissions than that in the unicast scenario.
- Due to the broadcast nature of wireless communication, all mobiles in the transmission range of a transmitter can simultaneously receive the transmitted packet. In the unicast scenario, only the one mobile (the destination of the packet) is interested in receiving the packet. In the multicast scenario, all the destinations belonging to the same multicast sessions are interested in the packet. Thus, one transmission can result in multiple successful deliveries in the multicast scenario.

Because of these differences, the multicast capacity of MANETs is different from the unicast capacity. The focus of this paper is to understand the scaling law of delay-constrained multicast in MANETs.

The scaling approach is introduced in [23], and has been extensively used to study the capacity of wireless ad hoc networks including both static and mobile networks. We consider an MANET consisting of \( n_s \) multicast sessions. Each multicast session has one source and \( p \) destinations. The wireless mobiles are assumed to move according to a two-dimensional independently and identically distributed (2D-i.i.d.) mobility model. Each source sends identical information to the \( p \) destinations in its multicast session, and the information is required to be delivered to all the \( p \) destinations within \( D \) time-slots. The main contributions of this paper include:

- Given a delay constraint \( D \), we prove that the capacity per multicast session is

\[
O\left( \min\left\{ 1, \left(\log p\right)\left(\log \left(n_s,p\right)\right)\right\} \right).
\]

We then propose a joint coding-scheduling algorithm achieving a throughput of

\[
\Theta\left( \min\left\{ 1, \left(\sqrt{\frac{D}{n_s p}}\right) \right\} \right).
\]

The algorithm is developed based on an information theoretical approach, where we exploits erasure codes to guarantee reliable multicast. The idea of exploiting coding has been used
in MANETs with unicast flows [13], [14], [16] and mobile sensor networks [24].

- We evaluate the performance of our algorithm using simulations. We apply the algorithm to the 2D-i.i.d. mobility model, random-walk model and random waypoint model. The simulations confirm that the results obtained form the 2D-i.i.d. model hold for more realistic mobility models as well.

Finally, we would like to remark that (a) Similar to the unicast scenario [1], the mobility significantly improves the throughput. While the multicast capacity of a static network is \( O\left(\sqrt{\alpha_k \log n_s} \right) \), our algorithm achieves a throughput of \( \Theta(1) \) when \( D = n_s \). (b) Our result again demonstrates the substantial benefit of using coding. While the algorithm in [22] achieves a throughput of \( \Theta\left(\frac{1}{p\sqrt{n_s}p \log p}\right) \) with an average delay \( \Theta(\sqrt{n_s}p \log p) \), our algorithm achieves a much higher throughput \( \Theta\left(\frac{1}{p \log p}\right) \) under a hard delay constraint \( \Theta(\sqrt{n_s}p \log p) \).

II. Model

We consider a mobile ad hoc network with \( n_s \) multicast sessions. Each multicast session consists of one source node and \( p \) destinations. Therefore, there are \( n \equiv n_s(p+1) \) mobiles in the network. A source sends identical information to all its destinations, and mobiles not belonging to the multicast session can serve as relays. All mobiles are positioned in a unit torus, where the left and right edges are connected, and top and bottom edges are also connected. We further assume the mobiles move according to 2D-i.i.d. mobility model [4] such that: (i) at the beginning of each time slot, a mobile randomly and uniformly selects a point from the unit torus and instantaneously moves to that point; and (ii) the positions of mobiles are independent across mobiles and across time slots.

Each mobile is equipped with a wireless antenna, and can communicate with another mobile within the transmission radius. We first assume that each mobile can adapt power and use an arbitrary transmission radius to obtain a general upper-bound on the delay-constrained multicast capacity. Then we propose a joint coding/scheduling algorithm that (i) achieves a near-optimal throughput, and (ii) requires only two transmission radii \( \{L_1, L_2\} \), where \( L_1 \) is for sending out information from sources, and \( L_2 \) is for delivering packets to the destinations.

We adopt the protocol model introduced in [25] for the wireless interference. Let \( \alpha_i \) denote the transmission radius of node \( i \), then a transmission from node \( i \) to node \( j \) is successful under the protocol model if and only if the following two conditions hold: (i) the distance between nodes \( i \) and \( j \) is less than \( \alpha_i \), and (ii) if mobile \( k \) is transmitting at the same time, then the distance between node \( k \) and node \( j \) is at least \( (1 + \Delta)\alpha_k \) (see Figure 1), where the \( \Delta > 0 \) defines a guard zone around the transmission. We adopt this protocol model because the mobiles are allowed to transmit with different powers (i.e., different transmission radii) under this model, which allows us to obtain a general upper bound on the multicast capacity of MANETs. Note that under this protocol model, the receiver of node \( i \) associates an exclusion region which is a disk with radius \( \Delta \alpha_i/2 \) and centered at the receiver of node \( i \). It has been shown in [25] that exclusion regions associated with successful transmissions should be disjoint from each other. We further assume that each successful transmission can transmit \( W \) bits per time-slot.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{The two transmissions can succeed simultaneously if the distance between node \( j \) and node \( k \) is larger than \( (1+\Delta)\alpha_k \) and the distance between node \( i \) and node \( h \) is larger than \( (1+\Delta)\alpha_i \).}
\end{figure}

\subsection*{Delay constraint:} We assume a hard delay constraint \( D \) in this paper. A packet is said to be successfully multicast if all \( p \) destinations receive the packet within \( D \) time slots after the source sends out the packet.

\subsection*{Multicast throughput:} Let \( \lambda \) denote the multicast throughput per multicast session and \( \Lambda_s[T] \) denote the number of bits that are successfully delivered to all the destinations of multicast session \( s \) up to time \( T \). A multicast throughput \( \lambda \) is said to be supportable under delay constrain \( D \) and loss probability \( \epsilon \) if there exists \( n_0 \) such that for any \( n > n_0 \), there exists a coding, routing, and scheduling algorithm such that every bit is successfully multicast with a probability at least \( 1 - \epsilon \), and

\[ \lim_{T \to \infty} \Pr\left(\frac{\Lambda_s[T]}{T} > \lambda, \forall i\right) = 1 \]

III. Upper Bound

In this section, we present an upper-bound on the multicast capacity of MANETs. Recall that multicast in MANETs is different from unicast in the following aspects:

- A source packet is destined to \( p \) destinations, so has a higher probability being deliverable compared to the unicast case, where a packet is said to be deliverable if at least one destination is within its transmission range.
- When a packet is transmitted, it can be received by all the destinations in the transmission range, which increases the efficiency of the transmission.

We say a packet is successfully delivered to a destination \( d_j \) if the destination receives the packet before the deadline expires. We denote by \( \Lambda_{d_j}[T] \) the number of successfully delivered bits to destination \( d_j \) up to time \( T \). Note that a successfully multicast packet is a packet that is successfully delivered to all its destinations, so \( \sum_{s_i} \Lambda_{s_i}[T] \leq \sum_{d_j} \Lambda_{d_j}[T] / p \).
Given a destination, we can classify the successfully delivered packets into two categories: (i) those packets that are transmitted directly from sources to destinations, and (ii) those packets that are delivered by the relays. In the following lemmas, we will bound the number of packets in each category. We define \( \Lambda^S_{d_j}[T] \) to be the number of bits directly transmitted from the source to destination \( d_j \), up to time \( T \). We further define \( \Lambda^R_{d_j}[T] \) to be the number of bits transmitted from relays to destination \( d_j \), up to time \( T \). We define \( \Lambda^S[T] = \sum_{d_j} \Lambda^S_{d_j}[T] \) and \( \Lambda^R[T] = \sum_{d_j} \Lambda^R_{d_j}[T] \).

**Lemma 1**: Assuming the 2D-i.i.d. mobility and the protocol models, we have

\[
E[\Lambda^S[T]] \leq 5k \log(n_s p) \left( WT \sqrt{\frac{32}{\Delta^2}} \sqrt{n_s p} \right) + \frac{16kWT}{\Delta^2} p(\log p) \log(n_s p).
\]  

**Lemma 2**: Assuming the 2D-i.i.d. mobility and the protocol models, we have

\[
E[\Lambda^R[T]] \leq 5k \log(n_s p) \left( \sqrt{\frac{32}{\Delta^2}} WT(p + 1) \sqrt{n_s D} \right) + \frac{16kWT}{\Delta^2} p(\log p) \log(n_s p).
\]

From the definition of the multicast capacity \( \lambda \), we have \( \lambda \leq (\Lambda^S[T] + \Lambda^R[T])/(Tn_sp) \) because a successful multicast requires \( p \) successful deliveries. To that end, we can derive the following theorem on the delay-constrained multicast capacity of MANETs.

**Theorem 3**: The delay constrained multicast capacity under the 2D-i.i.d. mobility and protocol model is

\[
\lambda = \begin{cases} 
0, & \text{if } D = o \left( \sqrt{\frac{\log(p)^2(n_s p)}{n_s}} \right); \\
\Theta(1), & \text{if } D = o \left( \frac{n_s}{\log(p)^2(n_s p)^2} \right); \\
O \left( \frac{\log(p)(\log(n_s p))}{n_s} \right), & \text{otherwise}.
\end{cases}
\]  

We omit the proofs due to space constraints. Interested readers can find the detailed proofs in [26].

**IV. JOINT CODING-SCHEDULING ALGORITHM**

In this section, we propose new algorithms that almost achieve the upper bound. We consider two different cases: \( n_s = \Theta(1) \) and \( n_s = \omega(1) \). For the first case, we show that a simple round robin scheduling algorithm achieves the upper bound. For the second case, we introduce a joint coding-scheduling algorithm that leverages erasure-codes and yields a throughput that differs from the upper bound by a poly-logarithmic factor.

### A. Case 1: \( n_s = \Theta(1) \)

When \( n_s = \Theta(1) \), a simple scheme is to let the sources broadcast their packets to all the mobiles in the network in a round-robin fashion. It is easy to see that under this simple algorithm, both throughput per multicast session and delay are \( \Theta(1) \).

### B. Case 2: \( n_s = \omega(1) \)

To approach the upper bound obtained in Theorem 3, in this subsection, we propose a scheme which exploits coding. In our algorithm, we code data packets into coded packets using rate-less codes — Raptor codes [27]. Assume that \( Q \) data packets are coded using the Raptor codes. The receiver can recover the \( Q \) data packets with a high probability after it receives any \((1 + \delta)Q\) distinct coded packets [27].

We use a modified two-hop algorithm introduced in [1], which consists two major phases — broadcasting and receiving. At the broadcasting phase, we partition the unit torus into square cells (broadcasting cells) with each side of length equal to \( 1/\sqrt{n_s} \). All sources use a transmission radius \( \sqrt{2}/\sqrt{n_s} \) in the broadcasting phase. To avoid interference caused by transmissions in neighboring cells, the cells are scheduled according to the cell scheduling algorithm introduced in [23] so that each cell can transmit for a constant fraction of time during each time slot, and concurrent transmissions do not cause interference. We assume each cell can support a transmission of two packets during each time slot. In the receiving step, the unit square is divided into square cells (receiving cells) with each side of length equal to \( 1/\sqrt{n_s} \). The transmission radius used in this phase is \( \sqrt{2}/\sqrt{p^2n_s} \). Note that the two transmission radii used in this algorithm are derived from a virtual channel model that is presented in our technical report [26].

Similar as in [16], we define four classes of packets in the network. We also define and categorize packets into four different types.

- Data packets: Uncoded data packets.
- Coded packets: Packets generated by Raptor codes.
- Duplicate packets: A coded packet may be broadcast to other nodes to generate multiple copies, called duplicate packets.
- Deliverable packets: Duplicate packets that are within the transmission range of one of the packet’s destinations.

**Joint Coding-Scheduling Algorithm**: We group every \( 2D \) time slots into a supertime slot. At each supertime slot, the nodes transmit packets as follows:

1. **Raptor encoding**: Each source takes \( D/n_s \) data packets, and uses Raptor codes to generate \( D \) coded packets.

2. **Broadcasting**: This step consists of \( D \) time slots. At each time slot, in each cell, one source is randomly selected to broadcast a coded packet to \( 9(p + 1)/10 \) mobiles in the cell (the packet is sent to all mobiles in the cell if the number of mobiles in the cell is less than \( 9(p + 1)/10 \)).
(3) **Deletion:** After the broadcasting phase, all nodes check the duplicate packets they have. If more than one duplicate packet belong to the same multicast session, randomly keep one and drop the others.

(4) **Receiving:** This step requires \( D \) time slots. At each time slot, if a cell contains no more than two deliverable packets, the deliverable packets are broadcast in the cell; otherwise, no node in the cell attempts to transmit. At the end of this step, all undelivered packets are dropped. The destinations decode the received coded packets using Raptor decoding.

**Theorem 4:** Assume that the delay constraint \( D \) is both \( \omega(\sqrt{n_s} \log(n_s p)) \) and \( o(n_s) \). For a sufficiently large \( n_s \), at the end of each super time slot, every source successfully multicast \( \frac{D}{500} \sqrt{\frac{D}{n_s}} \) packets with a probability \( 1 - \frac{1}{n_s p} \).

From the theorem above, we can see that the throughput per multicast session is

\[
\frac{D}{500} \sqrt{\frac{D}{n_s}} \times \frac{1}{2D} = \Theta \left( \sqrt{\frac{D}{n_s}} \right),
\]

which differs from the upper bound by only a poly-logarithmic factor.

**V. SIMULATIONS**

In this section, we verify our theoretical results using simulations. We implemented the joint coding-scheduling algorithm for different mobility models, including 2D-i.i.d. mobility, random walk model and random waypoint model. We consider an MANET consisting of \( n_s \) multicast sessions, and the mobiles are deployed in an unit square with \( n_s \) sub-squares. The random walk model and random waypoint model are defined in the following:

- **Random Walk Model:** At the beginning of each time slot, a mobile moves from its current sub-square cell to one of its eight neighboring sub-squares or stays at the current sub-square. Each of the actions occurs with probability 1/9.

- **Random Waypoint Model [28]:** At the beginning of each time slot, a mobile generates a two-dimensional vector \( V = [V_x, V_y] \), where the values of \( V_x \) and \( V_y \) are uniformly selected from \( [1/\sqrt{n_s}, 3/\sqrt{n_s}] \). The mobile moves a distance of \( V_x \) along the horizontal direction, and a distance of \( V_y \) along the vertical direction.

**A. Multicast throughput with different numbers of sessions**

In this simulation, the number of multicast sessions \( (n_s) \) varies from 200 to 1000, each multicast session contains \( p = 10 \) destinations, and the delay constraint is set to be \( 2D = 200 \) time slots. Figure 2 shows the throughput per \( 2D \) time slots of the three mobility models with different values of \( n_s \).

2In our simulations, we only counted the number of distinct packets delivered that are successfully delivered before their deadlines expire. We did not consider coding and decoding in our simulations.

Our theoretical analysis indicates that the throughput per \( 2D \) time slots is \( \Theta \left( 2D \sqrt{\frac{D}{n_s}} \right) \). To verify this scaling law, we plot \( \alpha \left( 2D \sqrt{\frac{D}{n_s}} \right) \) in Figure 2 with \( \alpha = 0.09 \). Our simulation result shows that the throughput per \( 2D \) time slots scales as \( 2D \sqrt{2D/n_s} \) under all three mobility models. Further, the 2D-i.i.d. mobility has the largest throughput and the random walk model has the smallest throughput. This is because the distance a mobile can move within one time slot is the largest under the 2D-i.i.d. model and is the smallest under the random walk model. Our result indicates that the throughput is an increasing function of the speed (the distance a mobile can move within a time slot).

![Fig. 2. Throughput per multicast session per 2D time slots with different \( n_s \).](image)

**B. Multicast throughput with different delay constraints**

In this simulation, we fixed \( n_s = 500 \) and \( p = 10 \), and varied \( D \) from 100 to 400 with a step size of 50. We also compared the throughput with function \( \alpha 2D \sqrt{2D/n_s} \) with \( \alpha = 0.075 \). For all three mobility models, the simulations confirm that the throughput per \( 2D \) time slots scales as \( 2D \sqrt{2D/n_s} \).

![Fig. 3. Throughput per multicast session per 2D time slots with different delay constrains](image)

**C. Multicast throughput with different session sizes**

In this simulation, \( n_s = 500 \), the delay constraint is \( 2D = 200 \), and \( p \) varies from 4 to 40 with a step size of 4. Figure 4
shows that the throughput is almost invariant with respect to $p$.

From the simulations above, we can see that the $\Theta(\sqrt{\frac{D}{n_s}})$ throughput is achievable not only under 2D-i.i.d. model, but under more realistic models such as random walk model and random waypoint model.

VI. Conclusion

In this paper, we studied the delay constrained multicast capacity of large-scale MANETs. We first proved that the upper-bound on throughput per multicast session is $O(\min\{1, (\log p)(\log (n_s/p))\sqrt{\frac{D}{n_s}}\})$, and then proposed a joint coding-scheduling algorithm that achieves a throughput of $\Theta(\min\{1, \sqrt{\frac{D}{n_s}}\})$. We also validated our theoretical results using simulations, which indicated that the results based on 2D-i.i.d. model are also valid for random walk model and random way point model. In our future research, we will study (i) the impact of mobile velocity on the communication delay and multicast throughput; and (ii) the delay constrained multicast capacity of MANETs with heterogeneous multicast sessions, e.g., different multicast sessions have different sizes and different delay constraints.

Acknowledgement: Research supported by the DTRA grant HDTRA1-08-1-0016.

References


